

# Do Tests of Capital Structure Theory Mean What They Say?

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## ABSTRACT

In the presence of frictions, firms adjust their capital structure infrequently. As a consequence, in a dynamic economy the leverage of most firms is likely to differ from the “optimum” leverage at the time of readjustment. This paper explores the empirical implications of this observation. I use a calibrated dynamic trade-off model to simulate firms’ capital structure paths. The results of standard cross-sectional tests on these data are consistent with those reported in the empirical literature. In particular, the standard interpretation of some test results leads to the rejection of the underlying model. Taken together, the results suggest a rethinking of the way capital structure tests are conducted.

RECENT EMPIRICAL RESEARCH IN CAPITAL STRUCTURE focuses on regularities in the cross section of leverage to discriminate between various theories of financing policy. In this research, book and market leverage are related to profitability, book-to-market, and firm size. Changes in market leverage are largely explained by changes in equity value. Past book-to-market ratios predict current capital structure. Firms seem to use debt financing too conservatively, and the leverage of stable, profitable firms appears particularly low. Even if firms have a target level of leverage, they move toward it slowly. Firms with low leverage react differently to external economic shocks from firms with high leverage.<sup>1</sup>

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<sup>1</sup> See Graham (2000) on conservatism in financing decisions; Titman and Wessels (1988), Rajan and Zingales (1995), Fama and French (2002), among others, on cross-sectional determinants; Fama and French (2002), Hovakimian, Opler, and Titman (2001), and Graham and Harvey (2001) on slow mean-reversion of debt ratios; Baker and Wurgler (2002) on the influence of past book-to-market ratios; Welch (2004) on the influence of changes in the market value of equity; Opler and

These findings are typically evaluated in terms of the comparative statics of various capital structure models. Each of these models is supported by some evidence and challenged by other evidence. This paper attempts to understand whether our interpretation of cross-sectional tests would change if firms optimally adjusted their leverage only infrequently.

The starting point for this study is a simple but fundamental observation. In a dynamic economy with frictions the leverage of most firms, most of the time, is likely to deviate from the “optimal leverage,” as prescribed by models of optimal financial policy, since firms adjust leverage by issuing or retiring securities infrequently, at “refinancing points.” Consequently, even if firms follow a certain model of financing, a static model may fail to explain differences between firms in the cross section since actual and optimal leverage differ. It has been long recognized that deviations from optimal leverage may create problems in interpreting the results of empirical research. For example, Myers (1984, p. 578) emphasizes that “any cross-sectional test of financing behavior should specify whether firms’ debt ratios differ because they have different optimal ratios or because their actual ratios diverge from optimal ones.”

This paper contributes to the literature by addressing exactly how the above problem has manifested itself in empirical studies. It also offers the intuition behind the ways in which this problem operates. I start by constructing a model of time-consistent optimal dynamic financing in the presence of frictions and then use the model to generate dynamic paths of leverage. The resulting cross-sectional data resemble data used in empirical studies along a number of dimensions. This allows me to replicate tests commonly used in such studies and ask to what extent the results are similar. My findings can be summarized as follows: (1) Cross-sectional tests performed on data generated by dynamic models can produce results that are profoundly different from their predictions for corporate financing behavior at refinancing points; (2) moreover, some results may lead to the rejection of precisely the model on which these tests are based, if the null hypothesis is formed on the basis of the relationships at the refinancing point; and (3) even a stylized trade-off model of dynamic capital structure with adjustment costs can produce results that are numerically consistent with some of those observed empirically.

The basic economic intuition behind these results lies in the observation that in any cross section firms are at different stages of their refinancing cycles, with almost no firms being at “date zero”, that is at a refinancing point. At any point in time, any two firms are likely to exhibit different reactions to the same shock even if these firms are identical from the date zero perspective. What causes their responses to differ are the histories of idiosyncratic shocks and the accumulations of past financial decisions. To relate any dynamic model to empirical studies, it is necessary to produce within the model a cross section of leverage ratios that is structurally similar to those that could have been studied

Titman (1994) on the reaction of highly leveraged companies to industry shocks; Korajczyk and Levy (2003) on their reaction to macro shocks; and Leary and Roberts (2005) and Kisgen (2006) on the frequency of refinancing.

by an empiricist. This also suggests that we may need to rethink empirical tests in this area and also highlights the importance of developing dynamic models of financing that are capable of delivering quantitative predictions.

While in principle the approach developed here is applicable to any theory of financial policy, a prerequisite for my analysis is a model that captures the dynamics of firms' financing behavior. Among the many existing explanations of capital structure, only the trade-off argument has a fully worked out dynamic theory that produces quantitative predictions about leverage ratios in dynamics. This theory suggests that firms arrive at their optimal capital structure by balancing the corporate tax advantage of debt against bankruptcy and agency costs. Using a trade-off model might seem questionable because the empirical evidence for this model is, at best, mixed. However, as I show in this paper, the data are more consistent with the dynamic trade-off theory than is traditionally thought, and so, *ex post*, using a trade-off model is more justified. I employ a standard state-contingent model of dynamic capital structure rooted in a trade-off argument. While several features differentiate the model from others in the field, the basic setup is widely used in the literature. In the model, firms are always on their optimal capital structure path, but, due to adjustment costs, they refinance only occasionally. Small adjustment costs can lead to long waiting times and large changes in leverage, a result consistent with the findings of Fischer, Heinkel, and Zechner (1989). Firms that perform consistently well re-leverage to exploit the tax shield of debt. Firms that perform poorly face a liquidity crisis and sell their assets to pay down debt. If their financial condition deteriorates still further, they resort to costly equity issuance to finance their debt payments and, when all other possibilities are exhausted, they default and ownership is transferred to debt holders. The benefit of having a more realistic model is that it allows for the assessment of the magnitude of economic effects.

I use the model in two ways. First, I determine the path of a firm's optimal financing decisions. This enables me to study the cross section of optimal leverage at times when firms change their leverage, that is, at refinancing points. Naturally, when firms are at their refinancing points, all the comparative statics predictions are in line with the predictions of the standard dynamic trade-off theory.

In the second stage of the analysis, I perform a number of cross-sectional tests on simulated dynamic data generated by the model. Several results stand out. First, the analysis highlights difficulties in interpreting the leverage–profitability relationship. According to the pecking order argument, more profitable firms reduce their dependence on costly external financing and thus decrease their leverage. According to the trade-off theory, higher profitability decreases the expected costs of distress and allows firms to increase their tax benefits by increasing leverage. Thus, an inverse relation between leverage and profitability, frequently found in the data and identified by Myers (1993) as perhaps the most pervasive empirical capital structure regularity, represents a significant failure of the trade-off model and is considered by some writers to be decisive in its rejection. In my model, expected profitability is positively related to leverage at the refinancing points. However, I show that in a dynamic

economy cross-sectional tests reveal a negative relation. The intuition is simple: With infrequent adjustment, an increase in profitability lowers leverage by increasing future profitability and thus the value of the firm. Similarly, a decrease in profitability increases leverage. For those firms that do not refinance, this results in a negative relation between leverage and profitability. Of course, in any period some firms refinance. In the simulations the subset of firms that do not refinance dominates and the cross-sectional relation between profitability and leverage is always negative. This effect is strengthened by the presence of systematic shocks in the firms' cash flow. In a number of cases, the magnitude of the coefficient is also consistent with empirical estimates.

Second, again using the model to simulate dynamic data, I replicate almost exactly the test recently conducted by Welch (2004). His main finding is that debt ratios are largely explained by past stock returns, implying that corporations do not readjust their debt levels to counteract the mechanistic effect of stock returns on leverage. This observation is important, not least because other determinants used in the literature are found to affect leverage, largely through stock returns. The results of the same regression tests conducted on the simulated data are numerically very similar to those obtained by Welch, suggesting that a stylized dynamic model with small adjustment costs may be consistent with these findings.

In addition, the framework can provide an explanation for the "debt issuance mystery" (Welch (2004)), that is, the apparent inconsistency between the passive behavior of managers in response to mechanistic changes in equity value and the overall active capital structure policies of corporations. Managers are passive, since, over a short horizon, there is almost a one-to-one relation between leverage and variables whose change is entirely determined by stock returns. These results obtain in the model since managers decide to change the firm's leverage based on changes in value over a long period, a variable that is largely orthogonal to recent equity returns. Thus, both in the cross section and consistent with empirical observation, changes in outstanding debt value are contemporaneously almost independent of the changes in market value of equity.

Third, since the behavior of the cross section in a dynamic economy is radically different from the comparative statics properties at the refinancing points, comparing empirical findings with the theoretical properties of leverage at refinancing points can be misleading. An example is provided by the debate on possible explanations for the so-called "low leverage puzzle." This refers to the observation that the median corporate debt-to-capital ratio in the United States over 1965 to 2000 averaged only 31.4%, with two out of five firms having an average debt-to-capital ratio of less than 20%,<sup>2</sup> while traditional trade-off models produce substantially higher numbers. That dynamic trade-off models imply

<sup>2</sup> Estimates are based on COMPUSTAT data on the book value of debt and market value of equity. The debt-to-capital ratio is defined as: COMPUSTAT data items d9 + d34 divided by d9 + d34 + d25 × d199. These are unadjusted figures. Adjusted figures (see Rajan and Zingales (1995)) would be lower.

excessively high leverage is not surprising in light of Merton Miller's (1977 p. 264) famous remark about "horse and rabbit stew." Bankruptcy costs are simply negligible compared to the tax benefits of debt. To explain the observed low levels of leverage, we need to better understand the factors that might offset the tax benefits. One proposed solution is to consider a dynamic framework. Studies by Goldstein, Ju, and Leland (2001) and Ju et al. (2003) show that if firms are allowed to increase debt in the future, they will opt for lower leverage initially. My results suggest that average leverage measured over time, that is, in "true dynamics," tends to be larger than the leverage measured at refinancing points. Empirical estimates of leverage should therefore be compared with the model estimates of leverage ratios obtained in a dynamic economy.

My paper builds on several strands of previous research. First, it shares with recent papers such as Leland (1998), Goldstein, Ju, and Leland (2001), and Ju et al. (2003) a theoretical framework in which the standard structural models of risky debt pricing are extended to incorporate dynamic financing behavior. These models follow, on the one hand, static capital structure models developed by, among others, Leland (1994) and, on the other, dynamic capital structure models developed by Fischer, Heinkel, and Zechner (1989), whose research is, in turn, based on insights by Kane, Marcus, and McDonald (1984, 1985). Fischer et al. (1989) are also the first to suggest that empirical studies of capital structure in the cross section might be more fruitful if the dependent variable were to reflect the behavior of leverage over time, for example, its range, rather than its value at a point in time.

My model most closely resembles that of Goldstein et al. (2001). A distinct feature of my model is that the firms whose value falls substantially face a prolonged period of turbulence instead of simply running up a large debt burden and then defaulting. The model is thus consistent with the empirical findings of Asquith, Gertner, and Scharfstein (1994), according to whom firms unable to service their debt obligations sell a fraction of their assets in order to pay down their debt.

The simulation approach followed here resembles, for example, Berk, Green, and Naik (1999), who focus on the cross-sectional relation between a firm's investment policy, systematic risk, and expected returns. To investigate cross-sectional patterns and regularities in their nonlinear dynamic economy they perform simulations, an approach I endeavor to replicate since my model also has strong nonlinearities. I calibrate firms' technology parameters to resemble, in a sense discussed later, the properties of samples of firms typically used in empirical studies. I then simulate data on firm values, leverage, etc. for dynamic economies and conduct a number of cross-sectional tests similar to those performed in the empirical literature. The evolution of firms' asset values, and therefore their financing decisions, are cross-sectionally dependent due to the presence of systematic shocks.

Several recent papers address issues similar to those considered here. Gomes (2001) examines the investment behavior of financially constrained firms. Using a related approach, he finds that standard investment regressions that use cash flow as an explanatory variable produce misleading results. Hennessy and

Whited (2005) show that a trade-off model can explain a number of empirically observed stylized facts by expanding the set of financial choices available to a firm. In their model, firms take into account internally generated funds when they choose the method of financing. Compared with my model, their model features endogenous investment, a richer tax environment, and more financial choices. On the other hand, it does not model adjustment costs for debt and also it does not allow for default. While their model and methods are substantially different from mine, the idea that it is essential to consider firms' behavior beyond date zero is central to both approaches. Using an empirical duration model, Leary and Roberts (2005) find that firms do rebalance their capital structure infrequently in the presence of adjustment costs. Theirs is a pure empirical paper, which derives its tests from the literature. However, it is closely related to my paper in that its hazard model estimation is justified by a model of infrequent adjustment. Their result lends empirical support to the main assumption of my analysis that imperfections make firms willing to refinance discontinuously. In addition, they find that the financing gap is an important determinant of the adjustment hazard, a phenomenon that a model with exogenous investment policy cannot explain.

The paper proceeds as follows. Section I presents and solves the model. Section II presents the simulation procedure and replicates a number of empirical tests on data generated from the model. Section III describes the robustness tests. Section IV concludes. The Appendix contains details of the simulation method.

## I. The Model

### A. The Case of an All-Equity Firm

My model employs a standard contingent claims framework to analyze an individual firm and is closely based on Goldstein et al. (2001). Specifically, I consider an economy populated by  $N$  firms, each of which is endowed with monopoly access to some infinitely lived project operating in continuous time. The value of each firm stems from a perpetual entitlement to the current and future income from the project (EBIT-generating machine). Income is divided between the net payout to claimholders and retained earnings. In line with many other models of capital structure, I retain the Modigliani and Miller assumption that the project's cash flows are invariant to financial policy.<sup>3</sup> Investment is financed by retained earnings where the latter are net of depreciation and result in book assets growing at a rate  $g$ . The growth of book assets is modeled similarly to Brennan and Schwartz (1984). The initial value of book assets,  $A_0$ , is equal to the initial value of the firm. The state variable in the model is the total time  $t$  net payout to claimholders,  $\delta_t$ , where claimholders include both insiders (equity and debt) and outsiders (government and various costs). The

<sup>3</sup> Several papers analyze interactions between financing and investment policy, including joint decisions on production and capital structure (Brennan and Schwartz (1984), Mello and Parsons (1992), Mauer and Triantis (1994)) and the effects of asset substitution (Leland (1998)).

evolution of  $\delta_t$  is governed by the following process under the pricing measure  $\mathbb{Q}$ .<sup>4</sup>

$$\frac{d\delta_t}{\delta_t} = \mu dt + \sigma dZ_t, \quad \forall t \geq 0, \delta_0 > 0, \quad (1)$$

where  $\mu$  and  $\sigma$  are constant parameters and  $Z_t$  is a Brownian motion defined on a filtered probability space  $(\Omega, \mathcal{F}, \mathbb{Q}, (\mathcal{F}_t)_{t \geq 0})$ . Here,  $\mu$  is the risk-neutral drift and  $\sigma$  is the instantaneous volatility of the project's net cash flow.

I assume that management always acts in the best interest of shareholders and, throughout the paper, I use the terms manager and equity holder interchangeably. To avoid further complication, the default-free term structure is assumed to be flat with an instantaneous after-tax riskless rate  $r$  at which investors may lend and borrow freely. The marginal corporate tax rate is given by  $\tau_c$ . The marginal personal tax rates,  $\tau_d$  on dividends and  $\tau_i$  on interest income, are assumed to be identical for all investors. Finally, all parameters in the model are assumed to be common knowledge.

Under these assumptions, consider a debtless firm with current cash flow  $\delta_0$ . The firm's current value is divided between equity and government, with the shareholders' value being equal to

$$E(\delta_0) = \mathbb{E}_{\delta_0} \left[ \int_0^\infty e^{-rs} (1 - \tau) \delta_s ds \right] = (1 - \tau) \frac{\delta}{r - \mu}, \quad (2)$$

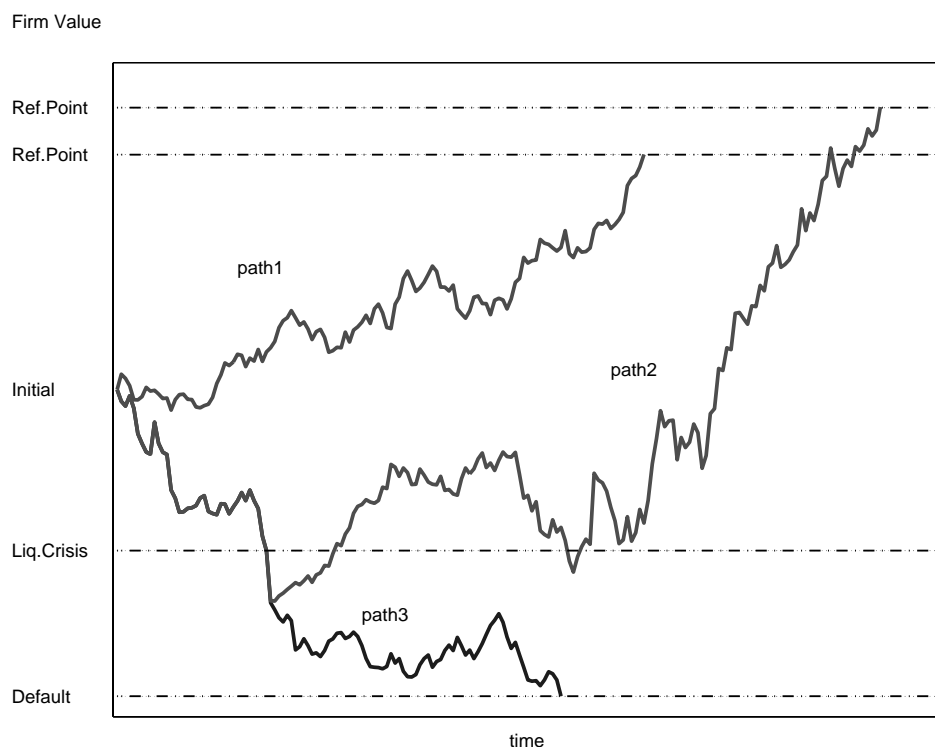
where  $\tau = 1 - (1 - \tau_c)(1 - \tau_d)$  and expectations, here and throughout the paper, are taken under the pricing measure  $\mathbb{Q}$ .

### B. The Case of a Levered Firm

Now, consider an otherwise identical firm whose management decides to choose the dynamic capital structure that will maximize the wealth of current equity. The fundamental driving force of the model is the inherent conflict of interest between the different claimholders since *ex ante* (prior to the issuance of debt) and *ex post* (after debt has been issued) incentives of equity holders are not aligned. Debt holders foresee the future actions of equity holders and value debt accordingly.

All corporate debt is in the form of a perpetuity entitling debt holders to a stream of continuous coupon payments at the rate of  $c$  per annum and allowing equity holders to call the debt at the face value at any time. To illustrate the model's structure in the presence of debt, Figure 1 shows a number of possible paths for the firm's value. At every date  $t$ , equity holders decide on their actions. As in Fischer et al. (1989), Leland (1998), and Goldstein et al. (2001), firms whose net payout reaches an upper threshold will optimally choose to retire

<sup>4</sup> Since I consider an infinite time horizon, some additional technical conditions on the Girsanov measure transformation (e.g., uniform integrability) are assumed here. In addition, the existence of traded securities that span the existing set of claims is assumed. Thus, the pricing measure is unique.



**Figure 1. Possible paths of firm value.** The figure shows possible model scenarios. Path 1 depicts a successful firm that refinances when firm value increases substantially. Paths 2 and 3 show firms that face a liquidity crisis. After selling assets and issuing equity the firm in Path 2 recovers and refinances when it reaches the upper restructuring threshold. The firm in Path 3 does not recover and equity holders decide to default when firm value is sufficiently low.

their outstanding debt at par and sell a new, larger issue to take advantage of the tax benefits associated with debt (path 1). Refinancing thus takes the form of a debt-for-equity swap. I refer to these thresholds as “refinancing points.”

For firms whose condition deteriorates sufficiently (paths 2 and 3), managers must take corrective action. Empirical research shows that firms often become insolvent on a flow basis but not on a stock basis. For such firms, the present value of future income exceeds their debt obligations but they experience a temporary liquidity crisis since fixed assets are a poor substitute for cash. In the model, this occurs whenever a firm’s cash flow is insufficient to cover its interest expense and thus the liquidity boundary is triggered for the first time at  $T_L$  whenever  $\delta_{T_L} < c$  and  $\delta_t \geq c$  for all  $t < T_L$ . This boundary closely resembles the definition of a financially distressed firm in Asquith et al. (1994) and a similar boundary is considered in Kim, Ramaswamy, and Sundaresan (1993). To resolve financial distress, firms are assumed to resort first to selling a fraction of assets to decrease their debt burden. In the Asquith et al. (1994) sample, the majority of firms do sell assets, with 18 out of 102 companies selling over 20% of their



assets. Additional assumptions capture several features of asset sales that are observed in practice. First, asset sales occur in discrete amounts: When firms divest assets, the transaction typically involves a significant fraction of their assets.<sup>5</sup> Second, asset sales are costly: Firms in financial distress realize less from asset sales than the present value of the cash flows from these assets since potential buyers are likely to be financially constrained, less well informed, and less experienced; sellers are time constrained and detach their human capital from sold assets. In other words, a discount can be viewed in terms of the traditional measure of liquidity (Shleifer and Vishny (1992)).

The corrective action is modeled as follows.<sup>6</sup> The firm sells a fraction  $1 - k$  of its assets immediately upon entering financial distress, which results in a reduction by a fraction  $w$  of the firm's outstanding debt:

$$(1 - q_A)(1 - k)V_L(1 - \tau) = \frac{(1 - w)D_0}{1 - q_{RC}}. \quad (3)$$

In (3)  $D_0$  is the par value of debt at the time of issuance and  $V_L$  is the present value of the project's future cash flows at time  $T_L$ . The parameter  $q_A$  represents the proportional costs incurred in selling assets, and  $\tau$  is the effective corporate tax rate on the asset sale.<sup>7</sup> Thus, the left-hand side is the after-tax income received by the firm as a result of the asset sale. Equality in (3) implies that all the proceeds are used to pay down debt. The proportional adjustment costs,  $q_{RC}$  are incurred by issuing/retiring debt.

An asset sale may lead to the firm's fortunes improving substantially, in which case it subsequently refinances (path 2), or it may provide the firm with only temporary breathing space (path 3). In the latter case, equity holders resort to equity issuance (effectively, negative dividends), as in earlier models. A number of empirical studies have shown that issuing equity is costly (Altinkilic and Hansen (2000), Hansen (2001), Corwin (2003)). In the model, the direct costs of external equity financing,  $q_E$ , are proportional to the amount issued.<sup>8</sup> Finally, equity holders optimally default if the firm's condition continues to worsen and the firm enters costly restructuring, which is modeled in reduced form. The absolute priority rule is enforced and all residual rights on the project are transferred to debt holders. Default costs are assumed to be a fraction  $\alpha$  of the value of assets upon default.

<sup>5</sup> Models of debt pricing also use "asset sales" or "asset liquidation" terminology (see, e.g., Black and Cox (1976)), but refer to the case of proportional asset liquidation that is equivalent to the net payout ratio being positive, since in those models cash flows originate exclusively via asset liquidation.

<sup>6</sup> Morellec (2001) also considers the effect of asset liquidity in a model of static optimal capital structure. Asset sales here differ from his case since they are conducted exclusively in financial distress, at prices that reflect a discount proportional to the firm's value at the time of sale, and are conducted in discrete amounts.

<sup>7</sup> The maximum price any buyer is willing to pay for these assets in the absence of frictions is  $(1 - k)V_L(1 - \tau)$ . I assume for simplicity that the buyer is unlevered. Note that since all firms face the same marginal tax rate,  $\tau$  is also the effective tax rate of an unlevered carbon copy of the firm.

<sup>8</sup> Acharya et al. (2002) introduce costly equity issuance in a structural model of credit spreads, but do not consider optimal leverage decisions.

The above discussion illustrates the model structure that gives rise to an optimal single-sided adjustment policy. To proceed with the valuation of financial claims, note that the model satisfies the so-called scaling feature since all costs are proportional to the value of the firm or its claims. In other words, at any refinancing point the firm is just a larger replica of itself. Therefore, I start by considering the values of equity and debt over one refinancing cycle (i.e., before the upper barrier is hit). These values, once debt is issued and before the liquidity barrier is hit, can be written as the sum of the present values of cash flows accruing to claimholders in four regimes: (i) while the firm is financially healthy, (ii) at the time the liquidity barrier is hit for the first time, (iii) in continuation after the barrier is hit, and (iv) in default. Thus, the values of equity and debt in one refinancing cycle at time  $t = 0$  are

$$\begin{aligned} E^R(\delta_0) = & \mathbb{E}_{\delta_0} \left[ \int_0^{T'} e^{-rs} (1 - \tau)(\delta_s - c) ds \right] \\ & + \mathbb{E}_{\delta_0} \left[ \int_{T_L}^{T''} e^{-rs} q \left( (1 - \tau)(k\delta_s - wc) - \tau_l wc \mathbf{1}_{[\delta_s < \delta_\tau]} \right) ds \right] \\ & + \mathbb{E}_{\delta_0} \left[ e^{-rT_B} \max \left[ (1 - \alpha) \int_{T_B}^{+\infty} e^{-rs} k(1 - \tau)\delta_s ds - wD_0, 0 \right] \middle| \phi_{LU}^B = 0 \right], \end{aligned} \quad (4)$$

and

$$\begin{aligned} D^R(\delta_0) = & \mathbb{E}_{\delta_0} \left[ \int_0^{T'} e^{-rs} (1 - \tau_i) c ds \right] \\ & + \mathbb{E}_{\delta_0} \left[ e^{-rT_L} | \phi_U^L = 0 \right] (1 - w)D_0 + \mathbb{E}_{\delta_0} \left[ \int_{T_L}^{T''} e^{-rs} (1 - \tau_i) wc ds \right] \\ & + \mathbb{E}_{\delta_0} \left[ e^{-rT_B} \min \left[ (1 - \alpha) \int_{T_B}^{+\infty} e^{-rs} k(1 - \tau)\delta_s ds, wD_0 \right] \middle| \phi_{LU}^B = 0 \right], \end{aligned} \quad (5)$$

where  $R$  stands for one refinancing cycle,  $T' = \min(T_L, T_U)$ , and  $T'' = \min(T_B, T_{LU})$ . The functions  $\phi_i^j$  take the value zero if event  $j$  occurs before event  $i$ , and one otherwise.

The first term in expression (4) is the present value of cash flows to equity holders when neither the liquidity barrier,  $\delta_L$ , nor the first refinancing barrier,  $\delta_U$ , have been reached. The second term is the present value of cash flows in continuation after the liquidity barrier has been hit and until either default occurs at time  $T_B$  or the second refinancing barrier,  $\delta_{LU}$ , is reached at  $T_{LU}$ . The function  $q(x)$  accounts for costly equity issuance and can be written as

$$q(x) = \begin{cases} x, & \text{if } k\delta_s > wc \\ (1 + q_E)x, & q_E > 0, \text{ otherwise.} \end{cases} \quad (6)$$

As in Goldstein et al. (2001), if corporate income,  $\delta_t$ , is sufficiently small, the firm loses part of its tax shelter and this results in a lower effective tax benefit,  $\tau - \tau_l$ . This reality is an important determinant of the leverage ratio at a refinancing point. The first and third terms in expression (5) are the net present values of payouts to debt holders before and after a liquidity crisis, respectively. The second term reflects the amount of debt purchased when assets are sold. In default equity holders receive either nothing or the residual after the remaining debt is repaid at its face value (the third term in (4) and the fourth in (5)).

The total value of a debt claim issued at date zero is thus

$$D(\delta_0) = D^R(\delta_0) + \mathbb{E}_{\delta_0}[e^{-rT_U} D_0 | \phi_L^U = 0] + \mathbb{E}_{\delta_0}[e^{-rT_{LU}} w D_0 | \phi_B^{LU} = 0]. \quad (7)$$

Equity holders make decisions taking into consideration what happens after refinancing occurs. The total value of all payouts to equity (except at refinancing points) is given by

$$\begin{aligned} E^D(\delta_0) &= E^R(\delta_0) + \mathbb{E}_{\delta_0}[e^{-rT_U} \gamma_U E^D(\delta_0) | \phi_L^U = 0] \\ &\quad + \mathbb{E}_{\delta_0}[e^{-rT_{LU}} \gamma_{LU} k E^D(\delta_0) | \phi_B^{LU} = 0] \end{aligned} \quad (8)$$

and the value of all debt issues is

$$\begin{aligned} D^D(\delta_0) &= D(\delta_0) + \mathbb{E}_{\delta_0}[e^{-rT_U} \gamma_U D^D(\delta_0) | \phi_L^U = 0] \\ &\quad + \mathbb{E}_{\delta_0}[e^{-rT_{LU}} \gamma_{LU} k D^D(\delta_0) | \phi_B^{LU} = 0], \end{aligned} \quad (9)$$

where  $\gamma_U$  and  $\gamma_{LU}$  are the proportions by which the net payout increases between two refinancing points if the liquidity barrier has or has not been hit, respectively.

Combining these values yields the total value of the firm that equity holders maximize at time  $t = 0$ , and after scaling, at each subsequent refinancing point:

$$F(\delta_0) = \frac{E^R(\delta_0) + (1 - q_{RC})D(\delta_0)}{1 - \gamma_U \mathbb{E}_{\delta_0}[e^{-rT_U} | \phi_L(U) = 0] - k \gamma_{LU} \mathbb{E}_{\delta_0}[e^{-rT_{LU}} | \phi_B(LU) = 0]}. \quad (10)$$

Thus, (10) states that managers maximize the sum of (1) the present value of the after-tax cash flows accruing to equity and (2) the present value of the after-tax income payments to all debt claims yet to be issued. Note that the total value takes into account the present value of future adjustment costs that will be incurred at subsequent refinancing points.

Equity holders choose the coupon and barriers to maximize the ex ante value of their claim, that is,

$$\mathbf{c}^* = \arg \max_{\{c, \gamma_U, \gamma_{LU}\} \in \mathbb{R}_+^3} [F(\delta_0)]. \quad (11)$$

An additional feature of realism, in which I follow Goldstein et al. (2001), is that the firm's financial decisions affect its net payout ratio. Empirically, higher reliance on debt leads to a larger net payout (see, e.g., Goldstein et al. (2001)). Here, for simplicity, I assume that the net payout ratio depends linearly on the after-tax coupon rate,

$$\frac{\delta}{V_t} = a + (1 - \tau_c) \frac{c}{V_0}, \quad (12)$$

where  $V_t$  is the present value of all future net payouts at time  $t$ .

To characterize the default threshold, note that equity holders balance the present value of future equity cash flows if they remain in control, with the cost of equity issuance that is incurred in this case. The relevant value of equity is  $E(\delta_t) = F(\delta_t) - D(\delta_t)$ , where the fact that the liquidity barrier has been hit is taken into account in calculating the value of claims. It is well known that this threshold satisfies the smooth-pasting condition:

$$\left. \frac{\partial E(\delta_t)}{\partial \delta_t} \right|_{\delta_t = \delta_B} = 0. \quad (13)$$

The full problem facing equity holders thus consists of solving (11) subject to (12) and (13). A closed-form solution to this problem does not exist, and thus standard numerical procedures are used.

### C. Comparative Statics

The purpose of this subsection is to compare the properties of firms' financial decisions at refinancing points in my model with the earlier literature. Table I summarizes the comparative statics of the main financial variables. The market leverage ratio,  $ML$ , is defined as the ratio of market value of debt ( $D(\delta_0)$ ) to total capital ( $F(\delta_0)$ ),

$$ML_0 = \frac{D(\delta_0)}{F(\delta_0)}. \quad (14)$$

Not surprisingly, many results are similar to the comparative statics results obtained by Leland (1994) for the static case (his table II for unprotected debt) and by Goldstein et al. (2001) for the dynamic case (their table 2). In particular, as expected, higher business risk, bankruptcy costs and a lower tax advantage all reduce optimal leverage. Contrary to the result given in Leland (1994), a higher risk-free interest rate unambiguously reduces leverage since the higher costs of borrowing more than offset the larger tax advantage to debt. Finally, an increase in the costs of asset sales and equity issuance also lowers borrowing.

The relation between the leverage ratio and adjustment costs exhibits an inverted U-pattern. Firms with either high- or low-cost access to external markets optimally prefer lower leverage than those with intermediate costs. This is because firms face a trade-off between the frequency of refinancing and the amount of borrowing. Firms with low costs prefer to rebalance frequently: As costs increase, the level of the refinancing boundaries rises (note that  $\delta_U$  and

Table I  
Comparative Statics of Financial Variables at the Refinancing Point

The table gives the comparative statics at the refinancing point of the following variables: the optimal market leverage ratio ( $ML$ ), bankruptcy boundary ( $\delta_B$ ), restructuring boundaries ( $\delta_U$  and  $\delta_{UL}$ ), total firm value ( $F(\delta_0)$ ), coupon rate ( $c$ ), and liquidity barrier ( $\delta_L$ ). The corporate tax rate is  $\tau_c$ ,  $\tau_d$  is the dividend tax rate,  $\tau_i$  is the interest income tax rate,  $r$  is the pre-tax risk-free interest rate,  $\sigma$  is the volatility of the firm's cash flow,  $\alpha$  is the fraction of asset value lost in bankruptcy,  $q_{RC}$  is the adjustment cost,  $q_A$  is the cost of selling assets in a liquidity crisis,  $q_E$  is the cost of equity issuance, and  $k$  is the fraction of asset value that remains after an asset sale.

Variable	Shape	Sign of Change in Variable for an Increase in:								
		$\tau_c, \tau_d$	$\tau_i$	$r$	$\sigma$	$\alpha$	$q_{RC}$	$q_A$	$q_E$	$k$
$ML$	Invariant to $\delta$	>0	<0	<0	<0	<0	>0, $q_{RC}$ small <0, $q_{RC}$ large	<0	<0	>0
$\delta_B$	Linear in $\delta$	>0	<0	<0	<0	<0	>0, $q_{RC}$ small <0, $q_{RC}$ large	<0	<0	>0
$\delta_U, \delta_{UL}$	Linear in $\delta$	<0	>0	<0	>0	<0	>0	>0	>0	<0
$F(\delta_0)$	Linear in $\delta$	<0	<0	>0	<0	<0	<0	<0	<0	>0
$c, \delta_L$	Linear in $\delta$	>0	<0	>0	<0	<0	>0, $q_{RC}$ small <0, $q_{RC}$ large	<0	<0	>0

$\delta_{UL}$  are increasing functions of  $q_{RC}$ ) and firms therefore borrow more, initially. As costs rise further, however, debt becomes less advantageous and is replaced by equity.

Rows 2 and 3 of Table I illustrate the behavior of the default and upper refinancing boundaries. The behavior of the default boundary, including its response to changes in the risk-free rate, is very similar to that of the leverage ratio. Higher costs of bankruptcy lead to a reduction in the level of the refinancing boundaries to offset the lower amount of borrowing. Higher volatility might also be expected to lower the level of the refinancing boundaries for the same reason, but it does not: Unlike bankruptcy costs, higher business risk increases both the expected costs of bankruptcy and the expected costs of refinancing in the future. The latter effect dominates and leads to the higher refinancing boundary.

The value of equity that managers maximize is negatively related to the tax rates on both corporate income and interest. This intuitive result is different from, for example, Fischer et al.(1989) and Leland (1994) since the state variable in their framework is the value of an unlevered firm and therefore tax benefits are accounted for as inflows of funds. The coupon level (and thus the liquidity boundary) is negatively related to firm volatility; the difference between “investment-grade” and “junk” firms observed by Leland (1994) disappears in a dynamic model. In Leland’s world, firms with very high levels of business risk optimally commit to pay sizable coupons since they expect a dramatic improvement in their fortunes with a nonnegligible probability. That would lead to a reduction in debt payments relative to firm value. In a dynamic world, they instead commit to refinancing at the same terms when their fortune improves.

Before turning to the investigation of the dynamic economy, it is worth pointing out briefly certain features that this class of models is not able to explain. First, these models have no endogenous investment and thus are unable to explain a number of observed phenomena, for example, the financing gap. Consequently, equity is never issued in good states of nature and the model cannot explain the negative relationship between current leverage and the past market-to-book ratio (Baker and Wurgler (2002)). Second, refinancing costs as modeled here are proportional to all debt outstanding and are not truly fixed costs. Therefore, the model cannot say anything about the relationship between firm size and capital structure.<sup>9</sup> Third, the model does not include an optimal cash policy, which in practice arises endogenously with the debt policy. Finally, the changes in firm value and contemporaneous cash flows are perfectly correlated, leading to an unrealistic correlation between some variables of interest. For example, the model produces too high a correlation between profitability and the market-to-book ratio.

## II. Capital Structure in a Dynamic Economy

The objective of this section is to investigate the cross-sectional properties of leverage ratios in a dynamic economy. Ultimately, I am interested in building a bridge between empirical research and the empirical hypotheses that the model delivers. The first step is to relate the leverage ratio and other variables of interest used in empirical studies to the variables used in the model. If firms adjust their leverage only periodically, *most firms most of the time will be optimally off their optimal leverage at a refinancing point*. Quite clearly, if an empiricist studies an economy generated by the model, the data would typically contain few “refinancing point” leverage ratios. Thus, to relate the model to empirical studies, it is necessary that *the model produces a cross section of leverage ratios that is structurally similar to those that would have been studied by an empiricist*.

The fact that using the implications of comparative statics may cloud inferences has been recognized for some time in studies of leverage mean-reversion and debt issuance (see, e.g., Hovakimian, Opler, and Titman (2001), Fama and French (2002)). If leverage deviates from its target substantially, an assertion that is supported empirically, then the response of firms to changes in economic conditions will not be in line with the predictions of comparative statics at refinancing points. Thus, I first study whether the cross-sectional relations in a dynamic economy are different from those at a refinancing point. Next, I use the data generated by the model to replicate a number of conventional cross-sectional studies of capital structure. This takes me to the crux of the existing empirical evidence. The two questions in which I am especially interested are whether my model can produce results that are qualitatively similar to those found in empirical research, and, if so, whether the empirical estimates could

<sup>9</sup> Kurshev and Strebulaev (2005) develop a dynamic trade-off model of capital structure with truly fixed costs of debt and investigate the ability of the model to explain the size–leverage relationship.

have been generated by the model with reasonable probability under a feasible set of parameters.

As in, for example, Berk et al. (1999), my model is highly nonlinear in a number of important parameters. As a result, individual dynamic leverage ratios, the main variable of interest, are difficult to obtain analytically. The complexity of dynamic effects in cross-sectional patterns of leverage means that it is impossible to identify the dynamic interaction between leverage and its determinants by performing a simple comparative statics exercise in dynamics. For example, a positive shock of a given magnitude can have different effects on firms in the same leverage group, leading to a complex interaction in the cross section, since some firms will refinance while others will not. Similarly, high leverage can be the result of either optimally high borrowing due to lower business risk or substantial refinancing costs and unsuccessful past returns.

Therefore, I use simulation to generate artificial data from the model. Simulation takes the solution to a dynamic problem faced by equity as given and does not involve any additional optimization. Since individual leverage ratios and some commonly used regressors are observable in the simulation, I am able to replicate a number of empirical research methods. In particular, I compare the cross-sectional properties of leverage in the simulated economy with those predicted from the comparative statics of leverage at refinancing points, the focus of most current theory, and then investigate the empirical hypotheses on the issues that have been the focus of many empirical studies. These issues include the average level of leverage in the economy, the cross-sectional relation between profitability and leverage, the mean-reversion of leverage ratios, and the impact of past stock returns on capital structure.

### A. Running Simulations

This section describes the simulation procedure. Simulations take the solution to the optimal capital structure at a refinancing point as given and do not involve any optimization mechanism. Technical details are given in the Appendix.

To begin, observe that while only the total risk of the firm matters for pricing and capital structure decisions, economy-wide shocks lead to dependencies in the evolution of the cash flow of different firms. To model such dependencies, shocks to their earnings are drawn from a distribution that has a common systematic component. Thus, cross-sectional characteristics of leverage are attributable both to firm-specific characteristics and to dependencies in the evolution of their assets. In particular, equation (1) may be rewritten as

$$\frac{d\delta_t}{\delta_t} = \mu dt + \sigma_I dZ^I + \beta\sigma_S dZ^S, \quad \forall t \geq 0, \quad \delta_0 > 0. \quad (15)$$

Here,  $\sigma_I$  and  $\sigma_S$  are constant parameters and  $Z_t^I$  and  $Z_t^S$  are Brownian motions defined on a filtered probability space  $(\Omega, \mathcal{F}, \mathbb{Q}, (\mathcal{F}_t)_{t \geq 0})$ . The shock to each project's cash flow is decomposed into two components, namely, an idiosyncratic shock that is independent of other projects ( $\sigma_I dZ^I$ ) and a systematic

(market-wide or industry) shock that affects all firms in the economy ( $\sigma_S dZ^S$ ). The parameter  $\beta$  is the systematic risk of the firm's assets, which I refer to as the firm's "beta," and systematic shocks are assumed independent from idiosyncratic shocks. The Brownian motion  $dZ$  in equation (1) is thus represented as an affine function of two independent Brownian motions,  $dZ = dZ^I + \beta dZ^S$ , and

$$\sigma \equiv (\sigma_I^2 + \beta^2 \sigma_S^2)^{\frac{1}{2}}. \quad (16)$$

At date zero all firms in the economy are "born" and choose their optimal capital structure. The comparative statics of the system at date zero (where all firms are at their refinancing points) is thus analogous to that described in Section C. For the benchmark estimation, I simulate 300 quarters of data for 3,000 firms. To minimize the impact of the initial conditions, I drop the first 148 observations, leaving a sample period of 152 quarters (38 years). I refer to the resulting data set as one "simulated economy." Using this resulting panel data set, I perform cross-sectional tests similar to those in the literature. The presence of a systematic shock makes cross-sectional relations dependent on the particular realization of the market-wide systematic component. Therefore, I repeat the simulation and the accompanying analysis a large number (1,000) of times. This allows me to study the sampling distribution for statistics of interest produced by the model in a dynamic economy.

In any period, each firm observes its asset value dynamics over the last quarter. If the value does not cross any boundary, the firm takes no action. It is important to stress that *it is optimal, under these conditions, for the firm to remain passive*. If its value crosses an upper refinancing boundary, it conducts a debt-for-equity swap, resetting the leverage ratio to the optimal level at a refinancing point, and so starting a new refinancing cycle. If the liquidity boundary is hit for the first time in the current refinancing cycle, asset sales are conducted in the same period. If the firm defaults, bondholders take over the firm and it emerges in the same period as a new firm with a new optimal leverage ratio. Observe that thus the procedure implies a constant population of firms in the economy. This is not an important restriction since the parameters for new firms would be drawn from the same sampling distribution as existing firms.

### B. Choice of Parameters

This section describes how firms' technology parameters and the economy-wide variables are calibrated to satisfy certain criteria and match a number of sample characteristics of the COMPUSTAT and CRSP data.<sup>10</sup> An important

<sup>10</sup> As becomes clear below, to compare the cross-sectional results of a date zero economy and a dynamic economy, I choose to present the scenario in which firm parameters are different. An alternative is to consider the case in which all firms are identical at refinancing points and thus any difference between them is accounted for only by random shocks. Similar qualitative results are obtained for this case, as shown in Section III.



caveat is that for most parameters of interest, there is little empirical evidence permitting precise estimation of sampling distributions or even their ranges. In addition, the model requires that all parameters be estimated as time-invariant. Overall then, the parameters used in the simulations must be regarded as ad hoc and approximate. There are two ways I deal with this problem. First, whenever possible (e.g., for tax rates), I use established empirical estimates. Second, and more importantly, I perform numerous robustness checks (see Section III). These robustness tests show that my results are not qualitatively affected by changing the parameters within a feasible range. Table II summarizes the descriptive information for the parameters described below.

### B.1. Firm Technology Parameters

The present values of the net payout and book assets at date zero are identical for each firm and scaled to 100. In the model, the rate of return on firm value is perfectly correlated with changes in earnings. In calibrating the standard deviation of net payout, I therefore use data on securities' returns. Firms differ in their systematic risk, represented by  $\beta$ . I obtain a distribution of  $\beta$  by running a simple one-factor market model regression for monthly equity returns for firms in the CRSP database having at least 3 years of data between 1965 and 2000 with the value-weighted CRSP index as the proxy for the market portfolio.

The distribution of firms' volatility is calibrated to match the parameters of the distribution of the standard deviation of rates of return on firm assets reported by Schaefer and Strebulaev (2005).<sup>11</sup> The mean and standard deviation of this distribution are 0.255 and 0.10, respectively. The standard deviation of the systematic shock,  $\sigma_S$ , is estimated as

$$\sigma_S = \sqrt{(1 - L_{av})^2 \sigma_E^2 + L_{av}^2 \sigma_D^2 + 2L_{av}(1 - L_{av})\sigma_{ED}}. \quad (17)$$

Here,  $\sigma_E$  is the volatility of monthly returns on the CRSP value-weighted equity return index,  $\sigma_D$  is the volatility of monthly returns on the CRSP 10-year T-note index over the period 1965 to 2000, and  $\sigma_{ED}$  is the covariance between equity and debt returns. Estimates of these parameters, 0.155, 0.081, and 0.023, respectively, are close to those reported by Campbell and Ammer (1993). Leverage,  $L_{av}$ , is computed from annual COMPUSTAT data for 1965 to 2000, averaging first for each year over firms and then over time. Leverage is defined as the ratio of book debt to the sum of book debt and market equity. The volatility of idiosyncratic shocks,  $\sigma_I$ , must be chosen to be consistent with the distribution of total risk. After considering a number of alternatives, individual shocks are assumed

<sup>11</sup> Note that the sample used in that paper is confined to firms that issue public debt. Faulkender and Petersen (2006) show that for firms without access to public debt markets, implied volatility is much higher. In robustness checks, I show that changing assumptions on the volatility distribution has an economically significant quantitative impact on average leverage ratios, though without affecting any qualitative cross-sectional results.

**Table II**  
**Parameter Values for Benchmark Simulations**

Listed are the values and sampling distributions chosen for all parameters required to simulate the benchmark case of the model.  $V_0$  is the present value of all future net payouts at time 0,  $A_0$  is the initial book value of firm assets,  $\beta$  is the systematic risk of the firm's assets,  $\sigma_E$  is the volatility of monthly returns on the CRSP value-weighted equity return index,  $\sigma_D$  is the volatility of monthly returns on the CRSP 10-year T-note index over the period 1965 to 2000,  $\sigma_{ED}$  is the covariance between equity and debt returns,  $L_{av}$  is average leverage computed from annual COMPUSTAT data for 1965 to 2000, averaging first for each year over firms and then over time, and defined as the ratio of book debt to the sum of book debt and market equity.  $\sigma_I$  is the volatility of idiosyncratic shocks,  $\sigma$  is the instantaneous volatility of the project's net cash flow,  $q_A$  is the proportional costs incurred in selling assets,  $q_{RC}$  is the proportional adjustment costs of issuing/retiring debt,  $q_E$  is the proportional direct costs of external equity financing,  $\alpha$  is the proportional restructuring costs,  $k$  is the fraction of assets that remains after an asset sale,  $\kappa$  defines the partial loss-offset boundary,  $g$  is the growth rate of book assets,  $a$  is the shift parameter in the net payout ratio estimation,  $RP_A$  is the asset risk premium,  $\tau_\kappa$  is the loss per dollar of full offset in the case of distress,  $\tau_c$  is the marginal corporate tax rate,  $\tau_d$  is the marginal personal tax rate on dividends,  $\tau_i$  is the marginal personal tax rate on interest income, and  $r$  is an instantaneous after-tax riskless rate.  $\mathcal{U}$  indicates uniform distribution.

Parameter	Distribution	Mean	Std. Dev.
$V_0$	Constant	100	
$A_0$	Constant	100	
$\beta$	Empirical	0.993	0.47
$\sigma_E$	Constant	0.155	
$\sigma_D$	Constant	0.081	
$\sigma_{ED}$	Constant	0.023	
$L_{av}$	Constant	0.314	
$\sigma_I$	$a_0 + a_1 \chi^2(n)$	0.22	0.107
	$\{a_0, a_1, n\} = \{0.05, \frac{1}{30}, 5\}$		
$\sigma$	Empirical	0.255	0.10
$q_{RC}$	$\mathcal{U}[0.0005, 0.0025] + 0.001s$	0.002	0.0006
$q_E$	$\mathcal{U}[0.02, 0.06] + 0.02s$	0.05	0.013
$\alpha$	$\mathcal{U}[0.03, 0.077] + 0.023s$	0.065	0.015
$q_A$	$\mathcal{U}[0.05, 0.183] + 0.067s$	0.15	0.043
$k$	$\mathcal{U}[0.6, 1]$	0.8	0.116
$\kappa$	$\mathcal{U}[0.7, 0.9]$	0.8	0.058
$g$	Constant	$\mu + RP_A$	
$a$	$\mathcal{U}[0.03, 0.04]$	0.035	0.003
$RP_A$	Constant	0.065	
$\tau_\kappa$	Constant	0.5	
$\tau_c$	Constant	0.35	
$\tau_i$	Constant	0.351	
$\tau_d$	Constant	0.122	
$r$	Constant	0.05	

to have a distribution with probability density function  $f(\sigma_I) \sim a_0 + a_1 \chi^2(n)$ . This distribution implies that projects with both low risk and very high risk are relatively common. A positive value of  $a_0$  also ensures that there will be no cash flows with negligible total risk.

Since the proportional costs of restructuring in default, adjusting leverage, selling assets, and issuing equity are all likely to be related to either the liquidity of firm assets and/or ease of access to external markets, all these costs are postulated to have a common covariance matrix. In particular, each cost,  $q_x$ , is drawn from the following distribution:  $q_x \sim \mathcal{U}[a_x, a_x + \frac{2}{3}(b_x - a_x)] + \frac{1}{3}(b_x - a_x)s$ , where  $a_x$  and  $b_x$  are bounds for the value of costs and  $s \sim \mathcal{U}[0, 1]$  is the common component. This formulation implies that 20% of each cost's value is due to the common component. This distribution is symmetric and its trapezoid probability density function implies that the values close to the boundaries are less likely to occur, while the values in a range around the mean are equally likely to occur.

For the proportional cost of restructuring in default,  $\alpha$ , the bounds  $a_x$  and  $b_x$  are assumed to be 0.03 and 0.10, respectively. Most of the empirical values reported in, for example, Weiss (1990) and Altman (1984) lie in this range. Recent evidence by Andrade and Kaplan (1998) suggests somewhat higher values. Leland (1994) uses a similarly defined cost of 0.5, Leland (1998) uses 0.25, and Goldstein et al. (2001) use 0.05.

Fischer, Heinkel, and Zechner (1989) and Goldstein et al. (2001) define adjustment costs,  $q_{RC}$ , in the same way and use a value of 1%. Datta, Iskandar-Datta, and Patel (1997) report total expenses of new debt issuance over 1976 to 1992 of 2.96%. Mikkelsen and Partch (1986) find underwriting costs of 1.3% for seasoned offers, and Kim, Palia, and Saunders (2003), in a study of underwriting spread over a 30-year period, find them to be 1.15%. This author's unreported calculation using the Fixed Income Securities Database (see Davydenko and Strebulaev (2007) for a description) over the period 1980 to 2000 suggests that the average underwriting and management spread is about 0.05% in yield, which is consistent, for example, with a proportional cost of 1% for a risk-free perpetuity when the risk-free rate is 5%. Note, however, that costs in this framework are proportional to the total amount of debt issued rather than to the incremental amount. I therefore choose substantially smaller adjustment costs, with a range of 0.05% to 0.35%, to be consistent with costs *per unit of new debt issued* of the order of 1%.

Proportional equity issuance costs are assumed to be distributed in the range (0.02, 0.08). Recent empirical research emphasizes that in initial public offerings, a simple 7% solution is used to settle underwriter costs (Hansen (2001)). The costs of seasoned equity offerings are likely to be smaller, however. Corwin (2003) reports a gross spread of 5.4% and direct expenses of 1.5%. In addition, there is evidence (Altinkilic and Hansen (2000)) that equity costs derive mainly from the variable component.

The costs of asset sales in a liquidity crisis are assumed to be distributed in the range (0.05, 0.25). These costs are, admittedly, enormously difficult to estimate. In one of the most elaborate empirical attempts to date, Pulvino (1998) estimates that on average these costs are around 14% for companies with an above median debt ratio. The fraction of assets that remains after an asset sale,  $k$ , is assumed to have a uniform distribution with support (0.6, 1). Asquith et al. (1994) report that, on average, companies sell 12% of their book assets.

The median level of asset sales among the firms that take visible steps in this direction is 48%.

The rate of net investment growth,  $g$ , is assumed equal to the expected growth rate of the firm's net cash flow. This is consistent with a finite nonzero expected market-to-book ratio at an infinite time horizon. It is also consistent with the fact that investment equal in magnitude to depreciation is needed to maintain the firm as a going concern. The net payout ratio increases with interest payments according to (12) and the parameter  $a$  depends, ultimately, on firms' price-earnings ratios and dividend policies. The range of the net payout ratio's value is between 0.03 and 0.04; the value of 0.035 is also used in Goldstein et al. (2001).

When the net payout flow is very small, the firm starts to partially lose its tax shelter. I model the partial loss offset boundary as  $\delta_\kappa = \kappa\delta_L + (1 - \kappa)\delta_B$ , where  $\kappa$  is uniformly distributed on (0.7, 0.9). It is assumed that, when the net payout is below  $\delta_\kappa$ , the loss per dollar of full offset is  $\tau_\kappa$ , where  $\tau_\kappa$  is set equal to 0.5. Note that this formulation assumes that full tax benefits are once again in effect when the firm comes out of distress.

### B.2. Economy-Wide Parameters

The corporate tax rate is assumed to be equal to the highest existing marginal tax rate,  $\tau_c = 0.35$ . To decide on marginal personal tax rates on interest income and dividend payments, I follow Graham (1999, 2000). In particular, Graham (1999) estimates  $\tau_i$  as 0.351 and  $\tau_d$  as 0.122 over the period 1980 to 1994. Thus, the maximum tax benefit to debt, net of personal taxes, is  $(1 - \tau_i) - (1 - \tau_c)(1 - \tau_d) = 7.8$  cents per one dollar of debt. In estimating tax rates, I ignore at least two important real-world features; the variability of tax rates both across firms and across time. Introducing time-varying taxes would destroy the scaling feature of the model. Since we do not know whether marginal firm-specific tax rates are correlated with firm characteristics, I choose to deal with firm-invariant tax rates.

The after-tax risk-free interest rate is 0.05. It is estimated as the mean 3-month Treasury bill rate over the period 1965 to 2000, multiplied by  $(1 - \tau_i)$ . Ibbotson Associates (1995) report an average annual equity risk premium of about 0.08 and expected default premium of about 0.01 for the postwar period. Using  $L_{av}$  (see the definition after equation (17)), the risk premium on the rate of return on firm assets is estimated in the region of 0.065.

### C. Preliminary Empirical Analysis

I now bring together the calibrated model with the results of comparative statics at the refinancing point and some empirical results from the literature. I use two definitions of leverage, both based on the market value of equity. The first, the market leverage ratio, has already been defined for date zero in (14); for any other period, it is defined analogously. Typically, however, market values of debt are not available and book values are used. I therefore introduce a second definition, the quasi-market leverage ratio, defined as the ratio of the

par value of outstanding debt to the sum of this par value and the market value of equity, that is,<sup>12</sup>

$$QML_t = \frac{D_0(\delta_t)}{D_0(\delta_t) + F(\delta_t) - D(\delta_t)}. \quad (18)$$

Typically, the difference between  $ML$  and  $QML$  is very small. For financially distressed firms it can be more substantial, however. Intuitively, these ratios reflect how the firm has financed itself in the past since both the par and market values of debt reflect decisions taken early in a refinancing cycle. To determine how close the firm is to financial distress, a flow measure that shows whether the firm can meet its debt payments is more relevant as firms may encounter distress at different levels of leverage. Therefore, I also consider the interest coverage ratio, which is defined as the ratio of the net payout to the coupon.

Table III summarizes the cross-sectional distribution of these various measures in both a dynamic economy and at the refinancing point. The average leverage ratio at the refinancing point is 0.26, compared with 0.37 in a similar model by Goldstein et al. (2001). The two main reasons for the difference are (1) the presence in my model of additional financial constraints such as liquidity crisis costs and (2) a lower tax advantage to debt since the tax rate on dividends that I use is smaller.

Of more importance, however, are the descriptive statistics for the dynamic economy. Means for dynamic statistics are estimated in a two-step procedure. First, for each simulated economy statistics are calculated for each year in the last 35 years of data. Second, statistics are averaged across years for each simulated economy and then over economies. To get a flavor of the impact of systematic shocks, for market and quasi-market leverage I also present minimum and maximum estimates over all economies. I begin by comparing the leverage statistics in the dynamic economy with those at refinancing points, where the impact of the dynamic evolution of firms' assets is ignored. What Table III shows is that leverage ratios in the dynamic cross section are larger than at refinancing points. An intuition for this observation is quite general: Unsuccessful firms tend to linger longer than successful firms who restructure fairly soon, especially so because firms who opt for higher leverage at refinancing points also choose a lower refinancing boundary.

Next I turn to a comparison with empirical data on leverage. Bernanke, Campbell, and Whited (1990) give the distribution of leverage for the 3 years 1986 to 1988. Their mean leverage ratio (0.33) is close to one given here (0.36). More interestingly, the right tail of my distribution mirrors theirs closely, suggesting that *a cross section of leverage ratios in a dynamic economy can replicate an empirically observed distribution, while the cross section at a refinancing point cannot*. Rajan and Zingales (1995) report, among other statistics,

<sup>12</sup> Where, in line with (7),  $D_0(\delta_t)$  is the par value of debt outstanding in the current refinancing cycle.

**Table III**  
**Descriptive Statistics**

The table reports descriptive statistics for the following variables: market leverage (*ML*), quasi-market leverage (*QML*), interest coverage ratio (the ratio of net payout,  $\delta$ , to coupon,  $c$ ), tax advantage to debt (i.e., the increase in firm value if the firm moves from no-leverage to its optimal leverage ratio, given by the formula  $\frac{F(\delta_t) - (1-\tau)V(\delta_t)}{(1-\tau)V(\delta_t)}$ , where  $F$  is firm value,  $\tau$  is the effective tax rate,  $V$  is the value of firm assets, and  $\delta_t$  is the level of cash flow at time  $t$ ). Ref. point refers to the case in which all firms are at their refinancing points. All other statistics are given for dynamics. One thousand data sets are generated, each containing 75 years of quarterly data for 3,000 firms. For each data set the statistics are first calculated for each year in the last 35 years of data and then are averaged across years. Finally, they are averaged over data sets. Min and Max give the minimum and maximum of the annual averages over the 1,000 data sets.

	Mean	Percentiles					Std. Dev.	N	
		1%	50%	90%	95%	99%			
<i>Market leverage, ML</i>									
Ref. point	0.26	0.04	0.27	0.40	0.43	0.50	0.10	3,000	
Average	0.36	0.06	0.34	0.56	0.66	0.87	0.16	3,000	
Min	0.30	0.06	0.29	0.46	0.53	0.76	0.13	3,000	
Max	0.43	0.07	0.41	0.71	0.80	0.94	0.20	3,000	
<i>Quasi-market leverage, QML</i>									
Average	0.37	0.06	0.35	0.59	0.70	0.91	0.17	3,000	
Min	0.31	0.06	0.29	0.47	0.56	0.82	0.14	3,000	
Max	0.44	0.07	0.42	0.74	0.84	0.96	0.21	3,000	
<i>Interest coverage ratio</i>									
Ref. point	3.98	2.01	3.22	5.74	7.80	17.83	3.24	3,000	
Average	3.08	0.69	2.64	4.78	6.08	11.26	2.35	3,000	
<i>Tax advantage to debt</i>									
Ref. point	0.05	0.02	0.05	0.07	0.07	0.08	0.01	3,000	
Average	0.04	0	0.04	0.07	0.08	0.09	0.02	3,000	

quasi-market leverage ratios. For 1991, the U.S. mean and median values are, respectively, 0.32 and 0.28, as compared with 0.37 and 0.35 in my model.<sup>13</sup>

Rajan and Zingales report a median interest coverage ratio of 2.41 (4.05) when deducting (not deducting) depreciation. For the former case, Bernanke, Campbell, and Whited (1990) report a mean value slightly above 5. Both results are similar in magnitude to the model values. The tax advantage to debt is calculated as the ratio of the difference between the current value of the firm and the after-tax value of unlevered assets to the after-tax value of unlevered assets. This ratio ranges between 0% and 10% with a mean of 5%. The gain in moving from no-leverage to optimal dynamic leverage, accounting for personal taxation, is comparable to the results on the net tax advantage of debt estimated by Graham (2000).

<sup>13</sup> To complement the comparison, I construct an empirical distribution of the quasi-market debt-to-capital ratio on COMPUSTAT data each year over 1965 to 2000. The 90<sup>th</sup> and 95<sup>th</sup> percentiles of the distribution are between 57% and 89%, and 62% and 92%, respectively. Footnote 2 defines the debt-to-capital ratio.

In summary, because firms at different stages in their refinancing cycle react differently to economic shocks of the same magnitude, the cross-sectional distribution of leverage is drastically different in dynamics and at the refinancing point, as is also the case for the other variables. Thus, care needs to be taken in using leverage properties at refinancing points in making any empirical claims.

#### *D. Cross-Sectional Regression Analysis*

This section examines several further applications of the approach developed in this paper. The main purpose of this exercise is to compare the results of regressions on simulated data to the results of empirical cross-sectional research. An example of the leverage–profitability relationship demonstrates that conventional methods can lead to the rejection of the model on which the data are based. An investigation of the impact of stock returns on leverage shows that this approach can be instrumental in providing an economic rationale for puzzling empirical observations.

##### *D.1. Leverage–Profitability Relationship*

This section shows that the dynamic relation between leverage and profitability is a particularly striking example for testing the credibility of empirical cross-sectional research. Profitability,  $\pi_t$ , is defined as the ratio of earnings before interest and taxes (the sum of the net payout ( $\delta_t$ ) and retained earnings (change in the value of book assets)) to the book value of assets in place,  $A_{t-1}$ :

$$\pi_t = \frac{\delta_t + \Delta A_t}{A_{t-1}}. \quad (19)$$

The trade-off theory predicts that a persistent increase in earnings leads firms to more extensive use of debt financing by increasing the tax advantage to debt and reducing the expected costs of distress and bankruptcy. This is reflected, for example, in a positive correlation (0.76) between leverage and profitability at the refinancing point.<sup>14</sup>

Why is the leverage–profitability relation singled out? First, as Myers (1993) points out, perhaps the most pervasive empirical capital structure regularity is the inverse relation between debt usage and profitability. Indeed, the relationship is one of several widely established results in the empirical capital structure literature.<sup>15</sup> More importantly, it is also one of a few, if not the only, cross-sectional relations that disentangles the (static and dynamic) trade-off

<sup>14</sup> Note that all changes in earnings in the model are persistent and thus firms with higher profitability at date zero expect to be more profitable in the future and opt optimally for higher borrowing.

<sup>15</sup> See, for example, Titman and Wessels (1988), Fama and French (2002), and Baker and Wurgler (2002). Rajan and Zingales (1995) establish that the inverse relationship holds for six out of seven developed countries and Booth et al. (2001) report that it also holds for most developing countries.

model and the various concepts associated with the pecking order idea, according to which, holding investment fixed, persistently higher profitability enables firms to use less leverage. For other determinants of leverage, either the predictions of both the pecking order argument and the trade-off theory are the same or the predictions of the various versions of the pecking order argument themselves differ.<sup>16</sup> The ambiguity attached to the impact of other determinants means that a consistently negative relation found between leverage and expected profitability is interpreted as major failure of the trade-off model.

I turn now to whether the cross-sectional leverage–profitability relation that my framework delivers is consistent with those reported in the empirical capital structure research. Recall that each simulated data set (economy) consists of 3,000 firms for 300 quarters and that economies differ because of a systematic shock. As described in Section A, I simulate 1,000 economies, dropping the first half of the observations in each economy. For each economy, I then conduct the regression tests outlined below. For each set of regressions, I report mean coefficients and *t*-statistics over all simulated economies and for several coefficients I also give the distribution.<sup>17</sup>

Table IV reports the results of the first set of experiments. Column 1 reports the regression for market leverage at the refinancing point and columns 2 to 4 report on simulated economies. Specifically, column 2 reports on attempts to replicate early empirical tests of capital structure (e.g., Bradley, Jarrell, and Kim (1984)) by performing an ordinary least squares (OLS) regression of quasi-market leverage, *QML*, at the end of the last year in each simulated economy against profitability and the constant “firm technology” parameters. Thus, the regressand and profitability are measured contemporaneously. Column 3 reports the results of the procedure that replicates the method implemented by Rajan and Zingales (1995) in which OLS regression of quasi-market leverage in year *t* is run against 4-year averages of the regressors over years (*t* − 4) to (*t* − 1), where year *t* is the last year in each economy.

Fama and French (2002) estimate “target leverage” using a two-step procedure. They first estimate year-by-year cross-sectional regressions and then use the Fama–MacBeth (1973) methodology to estimate time-series standard errors that are not clouded by the problems encountered in both single cross section and panel studies. The main problem with these methods stems from correlation in the regression residuals across firms and the presence of autocorrelation in the regression coefficients. In the simulated economy, correlation in the regression residuals exists because firm values are correlated via the systematic

<sup>16</sup> For example, both the pecking order and trade-off models predict that higher volatility of the firm’s cash flow is likely to lower the optimal amount of borrowing (see, e.g., Fama and French (2002)). Also, the static pecking order theory suggests that higher investment leads to higher borrowing when retained earnings are fixed, while the dynamic version predicts higher expected investment to decrease current debt so that the debt capacity is preserved for the future (see, e.g., Myers (1984)).

<sup>17</sup> Empirical studies include a number of variables (such as R&D) to control for firm heterogeneity that are clearly unnecessary in simulation. Conversely, the regressions on simulated data include firm-specific time-invariant parameters to control for firm heterogeneity.



Table IV  
Cross-Sectional Regressions

The table reports the results of cross-sectional regressions on the level of the quasi-market leverage ratio,  $QML$ . One thousand data sets are generated, each containing 75 years of quarterly data for 3,000 firms. Coefficients and  $t$ -statistics are means over 1,000 simulations. Independent variables are profitability ( $\pi$ ), volatility of cash flows ( $\sigma$ ), bankruptcy costs ( $\alpha$ ), asset sale costs ( $q_A$ ), and restructuring costs ( $q_{RC}$ ). The Ref. Point column gives the results obtained by running the regression at the refinancing point. The BJK, RZ, and FF columns report the results of regressions that replicate the empirical procedures used, respectively, by Bradley, Jarrel, and Kim (1984), Rajan and Zingales (1995), and Fama and French (2002). Each of the regressions is of the form:

$$QML^P = d_0 + d_1\pi^P + \mathbf{d}'\mathbf{x} + \epsilon^P,$$

where  $\mathbf{x}$  are firm technology parameters and  $P, P \in \{\text{BJK, RZ, FF}\}$ , refers to the method. For BJK and FF,  $QML^P = QML_{\text{end}}$  and  $\pi^P = \pi_{\text{end}-1}$ ; for RZ,  $QML^{RZ} = QML_{\text{end}}$  and  $\pi^{RZ} = \frac{1}{4} \sum_{m=\text{end}-1}^{\text{end}-4} \pi_m$ , where “end” is the last year in each data set. FF uses the Fama–MacBeth (1973) method, with the regressions run over the last 35 years of each data set and then averaged. The last three columns report additional information on the FF regression: the standard deviation of coefficients and  $t$ -statistics, and the 10<sup>th</sup> and 90<sup>th</sup> percentile values of these coefficients across simulations. BJK and RZ regression standard errors are standard. FF standard errors are Fama–MacBeth (1973) with the Newey–West correction.

	Ref. Point (1)	BJK (2)	RZ (3)	FF			
				Coeff. (4)	Std. Dev (5)	10% (6)	90% (7)
Constant	0.24 (22.29)	0.61 (29.36)	0.58 (28.06)	0.62 (34.02)	0.06 (21.03)	0.55 (14.60)	0.71 (62.22)
$\pi$	5.88 (30.95)	−0.76 (−6.81)	−0.47 (−4.18)	−0.78 (−7.47)	0.58 (4.20)	−1.53 (−12.46)	−0.22 (−3.54)
$\sigma$	−0.78 (−91.22)	−0.39 (−13.05)	−0.38 (−12.55)	−0.40 (−26.79)	0.04 (13.35)	−0.45 (−43.01)	−0.36 (−13.64)
$\alpha$	−0.32 (−7.30)	−0.47 (−2.28)	−0.47 (−2.28)	−0.47 (−10.25)	0.11 (5.66)	−0.60 (−17.50)	−0.33 (−4.84)
$q_{RC}$	3.67 (3.54)	−3.16 (−0.64)	−3.14 (−0.63)	−3.28 (−3.09)	2.53 (2.90)	−6.56 (−6.48)	−0.11 (−0.08)
$q_A$	−0.17 (−10.94)	−0.28 (−3.93)	−0.29 (−3.93)	−0.29 (−16.53)	0.04 (8.67)	−0.34 (−26.59)	−0.24 (−8.48)
$R^2$	0.89 (1)	0.07 (1)	0.07 (1)	0.08 (35)	0.01 (35)	0.06 (35)	0.10 (35)
$N$	3,000	3,000	3,000	3,000	—	3,000	3,000

shock and the slopes are also autocorrelated because leverage is a cumulative outcome of past idiosyncratic shocks. I choose to report Fama–MacBeth (1973) standard errors with the Newey–West correction.<sup>18</sup> The results are in column 4 of Table IV.

<sup>18</sup> Following the results of Petersen (2005), I used a number of methods to estimate standard errors. Petersen (2005) finds that Fama–MacBeth standard errors underestimate true errors even after the correction for autocorrelation. I also estimate Rogers (1993) standard errors clustered by firm and by time. Unreported, Rogers standard errors clustered by firm are smaller than the

To summarize, each of the regressions above can be written as

$$QML^P = d_0 + d_1\pi^P + \mathbf{d}'\mathbf{x} + \epsilon^P, \quad (20)$$

where  $\mathbf{x}$  are firm technology parameters and  $P, P \in \{\text{BJK}, \text{RZ}, \text{FF}\}$ , refers to the method. For example,  $QML^{\text{RZ}} = QML_t, \pi^{\text{RZ}} = \frac{1}{4} \sum_{m=t-1}^{t-4} \pi_m$ , and so forth.

For refinancing points, column 1 in Table IV reports that a 1% increase in expected profitability increases target leverage by 5.88% and a 1% increase in the firm's business risk produces a 0.78% reduction in leverage. The effect of bankruptcy and distress costs is smaller in absolute magnitude, demonstrating again that by themselves, these costs are not sufficient to offset the tax advantage to debt in the dynamic trade-off model. Insignificance of adjustment costs is due to their nonmonotonic relation to leverage.

Columns 2 to 4 show that *consistent with empirical findings* the relationship between profitability and leverage can be negative in a dynamic economy *even* for the trade-off model. The results of columns 2 to 4 are roughly similar, consistent with Fama–French's observation that their results are mainly supportive of previous findings, demonstrating that subtle variation in definition of leverage and profitability or in particular empirical method does not matter much. The Fama–MacBeth estimates with the Newey–West correction produce statistically negative average slopes.

Note that this result is of particular importance: *An empiricist would be likely to interpret this finding as evidence in favor of the pecking order argument and contrary to the predictions of the dynamic trade-off model.* Yet, we know that firms in the simulated economies do indeed follow the prescription of the dynamic trade-off theory. Why, in this case, is the profitability coefficient significantly negative in the dynamic economy? An increase in profitability affects future profitability and thus the value of the firm. But while an increase in the value of the firm always lowers leverage, it does not necessarily lead to refinancing in a world with infrequent adjustment. Note that under the model, the target leverage for any firm is constant, and so the observed positive relation between leverage and profitability at the refinancing point is purely a cross-sectional effect. The negative relation is at the individual firm level since higher profitability lowers the current leverage of an individual firm, unless it refinances in that period. The negative coefficients in Table IV imply that the effect at the individual firm level dominates in the simulated data. The presence of a systematic shock magnifies this effect.

That the presence of frictions may complicate inferences has been recognized in a number of previous studies. For example, Fama and French (2002) note

Fama–MacBeth ones with adjustment for time-varying regressors and larger for time-invariant regressors. Under all methods, the results are statistically significant. Fama and French assume for simplicity that the standard errors of the average slopes should be multiplied by a certain factor to account for autocorrelation before judging the significance of a variable. Unreported results demonstrate that the average coefficient on profitability is autocorrelated and behaves like an AR(1) process with observed maximum of about 0.75 and thus (see Fama and French (2002, p. 12)) the corresponding multiplication factor is 2.5.

that their result may overstate the long-term relation between leverage and profitability by picking up transitory variation in leverage rather than variation in target leverage. This would make it difficult to disentangle the dynamic trade-off and pecking order models since a negative coefficient may be the result of the transitory component, pecking order behavior, or both. It is therefore instructive to look at the size of the coefficient in the simulated data to judge whether a dynamic trade-off model can give rise to values that are similar to those found empirically.

The population mean of the profitability regression coefficient is above those found by previous researchers. Profitability coefficients reported by previous studies include  $-0.90$  (Fama and French (2002)),<sup>19</sup>  $-0.6$  (Rajan and Zingales (1995)), and  $-0.61$  (Baker and Wurgler (2002)). However, my estimate of  $-0.78$  for the Fama and French type of regression is simply the population mean across all economies. To gauge the likelihood of obtaining estimates in the range of  $-0.6$  to  $-0.9$  under the model, I examine the distribution of the profitability coefficient. Columns 6 and 7 of Table IV report its 10<sup>th</sup> and 90<sup>th</sup> percentiles. All the coefficients are negative and most of them are significant. Thus, under the chosen set of parameters, the reported empirical estimates can be consistent with the value of the coefficient.

There are several possible ways in which this result may be qualified. First, the parameter set may be unrepresentative because, for example, I do not allow for correlation between volatility and distress/bankruptcy costs. Indeed, in a number of robustness checks the resulting coefficient, while still negative, is substantially smaller in magnitude. In particular, smaller restructuring costs and more widely dispersed “betas” result in a smaller coefficient. For many other parametrization changes, the result is unchanged or stronger.

Second, I use the leverage ratio based on the market value of equity. Fama and French (2002) argue that the profitability–leverage relation holds theoretically only for book leverage. In empirical regressions, however, the values of the slope are very similar. Therefore, while for book leverage the result is likely to hold under a broader set of conditions than for market leverage, it is unlikely that this drives the observed difference.

Third, in my model as well as most dynamic models of optimal structure, the investment process is independent of the process that determines the leverage ratio. In deriving the book value of assets, I make an assumption that book assets grow at a rate equal to the growth rate of the net payout under the actual distribution—the only rate under which the market-to-book ratio has a finite nonzero expected value at an infinite time horizon. I choose a conservative value of one for an initial market-to-book ratio since my firms may be characterized as value firms. A decrease in the book value of assets, however, would lead to an

<sup>19</sup> Fama and French report several profitability coefficients, ranging from  $-0.42$  to  $-0.96$ , since they study both book and market leverage, divide the sample of firms into two groups (dividend payers and nonpayers), and include in some regressions a simultaneously estimated target payout ratio. The coefficient of  $-0.9$  is for the regression on market leverage for dividend payers, not allowing for the target payout ratio.

increase in profitability without changing quasi-market leverage and in turn to a decrease in the magnitude of the profitability–leverage coefficient.

In a nutshell, notwithstanding all these caveats, the analysis here and the robustness tests in Section III suggest that, at the very least, the model is able to explain a substantial part, if not most, of the negative relation between profitability and leverage. All other coefficients in Table IV have the same sign in dynamics as at the refinancing points, although the magnitude of the volatility coefficient is smaller in the dynamic context. Adjustment costs become significant in a dynamic economy since their increase leads to higher refinancing boundaries, and thus the longer average waiting times between successive adjustments, and correspondingly the change in leverage, is larger.

### D.2. Leverage and Stock Returns

In a recent paper, Welch (2004) obtains empirical results that to some extent parallel those presented here. Welch's main finding is that U.S. corporations do not change their capital structure to offset the mechanistic effect on leverage of changes in their stock price. The ongoing debate surrounding these results is motivated by at least two observations: the conventional firm characteristics lose their significance in the presence of lagged equity returns and several empirical stylized facts that remain largely unexplained.

As I emphasize above, the absence of a response by the firm to these mechanistic changes in leverage may, indeed, be optimal in the presence of adjustment costs. It is therefore instructive to investigate the extent to which the mechanistic effect observed by Welch is reflected in my dynamic economies. To this end, I replicate, again using simulated data, the regression test that he performs on the COMPUSTAT data set (Welch (2004), table 3). For each year  $t$ , I run a cross-sectional regression of the level of the market leverage ratio against (1) the implied market debt ratio,  $IDR_{t-k,t}$  in Welch's notation, that is, what the market leverage ratio would have been if the firm had not issued any securities between years  $t-k$  and  $t$ , and (2) the actual observed quasi-market leverage ratio in year  $t-k$ ,  $QML_{t-k}$  in my notation:

$$QML_t = f_0 + f_1 IDR_{t-k,t} + f_2 QML_{t-k} + \epsilon. \quad (21)$$

The implied debt ratio shows the response of leverage only to changes in equity. Thus, if the coefficient  $f_1$  is equal to 1, firms do not readjust at all. Alternatively, a value of  $f_2$  equal to 1 would imply that firms perfectly offset any change in equity.

The estimated regression (21) is identical to that studied by Welch. The only point of departure between my simulations and the empirical procedure followed by Welch is that the number of firms in the empirical study varies across years while in the simulations the number of firms is fixed.

To be precise, I compute the average of the time series of cross-sectional regression coefficients *à la* Fama–MacBeth. Then, as usual, the results are averaged over many simulated economies. Table V shows that for all four choices

Table V  
Leverage and Stock Returns

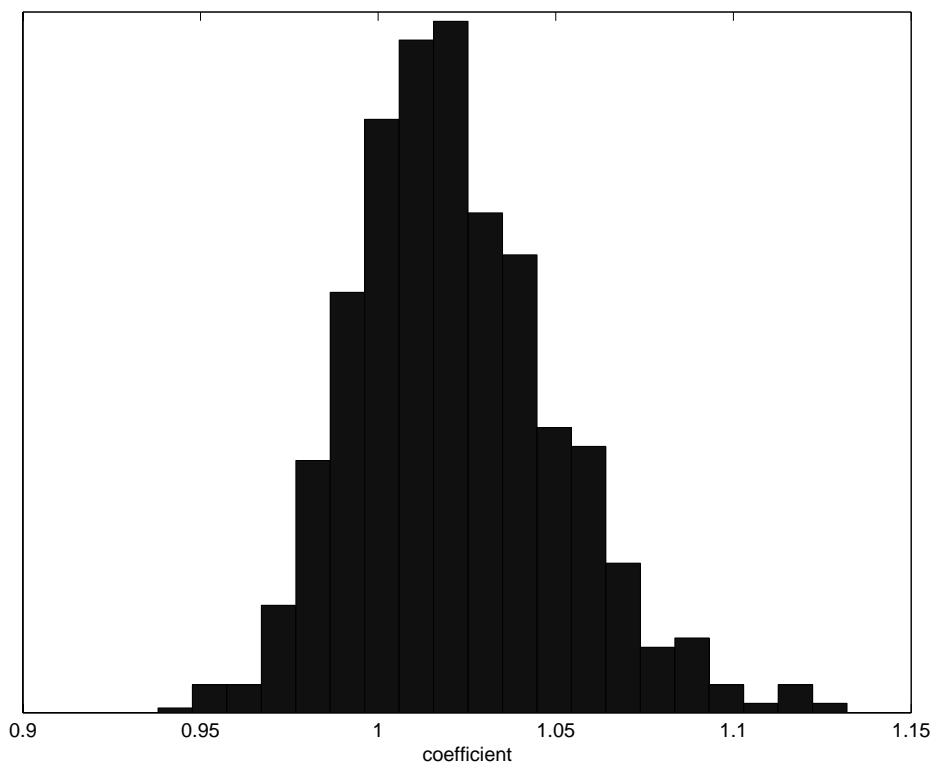
The table reports the results of the following cross-sectional regressions on the level of the quasi-market leverage ratio,  $QML$ .

$$QML_t = f_0 + f_1 IDR_{t-k,t} + f_2 QML_{t-k} + \epsilon.$$

Independent variables are the implied debt ratio ( $IDR_{t-k,t}$ ) and lagged quasi-market leverage ratio ( $QML_{t-k}$ ). Coefficients and  $t$ -statistics in Panel A are means over 1,000 simulations. Row 1 of Panel B reports Welch's (2004) estimates of the IDR coefficients. Other rows report the mean and the 5<sup>th</sup> and 95<sup>th</sup> percentiles of my estimates. One thousand data sets are generated, each containing 75 years of quarterly data for 3,000 firms. For each data set the above regressions are run over the last 35 years of data and then averaged. Standard errors are Fama–MacBeth (1973) with the Newey–West correction.

	<i>k</i> Years			
	1	3	5	10
Panel A				
Constant	0.034 (34.241)	0.088 (41.942)	0.130 (45.289)	0.199 (46.679)
$IDR_{t-k,t}$	1.022 (92.327)	0.886 (75.677)	0.781 (71.356)	0.592 (53.735)
$QML_{t-k}$	−0.105 (−7.971)	−0.095 (−5.868)	−0.089 (−5.445)	−0.063 (−5.028)
$R^2$	0.926 (37)	0.802 (35)	0.698 (33)	0.502 (28)
$N$	3,000	3,000	3,000	3,000
Panel B: IDR Coefficients				
Welch	1.014	0.944	0.869	0.708
This paper	1.022	0.885	0.780	0.591
5%	0.980	0.839	0.735	0.552
95%	1.072	0.935	0.825	0.628

of  $k$ , between 1 and 10 years, the results appear to conform closely to those obtained by Welch. In particular, the slope of nearly one for the implied debt ratio for the 1-year regression (average slope of 1.014 in Welch and 1.022 in the model) indicates that financing behavior in the short term is almost passive; in other words, corporations do not react to changes in the value of equity by adjusting their leverage. Figure 2 demonstrates that the coefficient over the 1-year horizon, obtained by Welch for the implied debt ratio, is well within the observed frequency of average coefficients in the model, and Table V shows that my model produces slightly lower estimates for longer horizons. Overall, I find that my model does not reject Welch's coefficient on the implied debt ratio over a short horizon and that the term structure patterns of the coefficients are also similar. The simulations clearly show that a model with small adjustment costs can produce results on the persistence of leverage that are consistent with those observed in reality.



**Figure 2.  $IDR_{t-1,t}$  coefficient.** The figure shows the distribution of the implied debt ratio coefficient in regressions of the quasi-market leverage level against the implied debt ratio and last year's leverage (Welch (2004)).

There is one particular feature that deserves special attention. Welch (2004) points out that while corporations do change their capital structure, their motives “remain largely a mystery” given that the mechanistic effect of the change in equity value is not offset. The same apparent puzzle is observed in my framework. A coefficient close to 1 might be interpreted as extreme passivity on the part of shareholders in their debt decisions. At the same time, about 12% of firms refinance every year in the model, consistent with the observation of changing capital structure. In fact, my model provides a simple explanation of this “puzzle” since firm “passivity” in the Welch (2004) sense also obtains if firms issue debt quite frequently, but the contemporaneous cross-sectional covariance between new debt issues and equity returns over the chosen period is zero. This is exactly what happens in the model since managers issue debt in response to a factor that is largely orthogonal to short-term equity returns, namely, long-term past stock returns. The model provides an additional insight: If the covariance between changes in outstanding debt and equity returns over  $t - k$  to  $t$  is weakly positive, then the coefficient on  $IDR_{t-k,t}$  increases slightly. For  $k = 1$  year, the empirically observed covariance is indeed weakly positive,

explaining why the coefficient slightly exceeds 1. Observe that in the model, while the equity return over the last year does not trigger debt issuance by itself, debt will be issued only if equity returns are positive (otherwise the refinancing barrier would not be reached). In addition, debt reduction due to a liquidity crisis occurs only if the last-period equity return is negative, leading again to weakly positive covariance and a 1-year coefficient slightly exceeding 1, on average. It also provides an explanation for why, over a long horizon, my coefficients are smaller than Welch's: Firms may issue debt for reasons related to investment opportunities that can be positively related to changes in equity value.

Thus, managers react to long-term as opposed to short-term shocks. In the present model, each firm takes into account what has happened since the start of the last refinancing cycle. The thrust of the economic intuition is that in a dynamic economy refinancing cycles of firms overlap. By forcing explanatory variables to be taken at one point in time for the whole cross section, the analysis always tends to suggest passivity. It is thus tempting to suggest that adjustment costs are entirely responsible for these stylized facts. However, as Welch himself points out, there are some drawbacks to this explanation: (1) Direct transaction costs are small; (2) readjustment patterns are similar across firms while transaction costs are very different; and (3) firms do not seem to lack the inclination to actively adjust capital structure, but they seem to lack the proper inclination to readjust when equity value changes. My analysis sheds light on some of these concerns. First, even small transaction costs can lead to stickiness in the firm's debt policy. Robustness checks in Section III show that even taking a highly conservative estimate of transaction costs leaves the results essentially unchanged. Second, in the model, debt issuance costs are smaller than equity issuance costs, thus the firms that reduce debt when they are in distress experience relatively higher transaction costs. In other words, after substantially negative equity returns firms face higher transaction costs. However, these firms are no more eager to readjust. Third, as I explained above, the framework accounts for both the capital structure activity and the unwillingness to readjust in response to past equity returns. At the same time, at least two issues raised by Welch (2004) cannot be addressed satisfactorily in the present framework. First, there is no difference between small and large firms, and second, no richer set of debt instruments is allowed that would enable corporations to lower transaction costs.

### *D.3. Changes in Leverage and Mean-Reversion*

I turn next to the question of the extent to which leverage is mean reverting in my model. Table VI summarizes estimates of a number of partial adjustment models, where the dependent variable in all cases is the annual change in the quasi-market leverage ratio. Columns 1 and 2 of the table report the results of a two-stage cross-sectional regression estimation. In the first stage, target leverage,  $TL$ , is estimated using equation (20); the resulting value is then used in the regression for changes in leverage:

**Table VI**  
**Cross-Sectional Regressions for Leverage Changes**

The table reports the results of the following Fama–MacBeth (1973) cross-sectional regressions on changes in the quasi-market leverage ratio,  $QML_t - QML_{t-1}$ :

$$QML_t - QML_{t-1} = h_0 + h_1 TQML_{t-1} + h_2 QML_{t-1} + h_3 X_{t-1} + \epsilon.$$

Independent variables are the target quasi-market leverage ratio ( $TQML_{t-1}$ ), past leverage ( $QML_{t-1}$ ), implied debt ratio adjustment ( $IDR_{t-1,t} - QML_{t-1}$ ), profitability ( $\pi_t$ ), change in profitability ( $\Delta\pi_{t-1} = \pi_{t-1} - \pi_{t-2}$ ), and the cross-term ( $(\pi_{t-1} \times (IDR_{t-1,t} - QML_{t-1}))$ ). One thousand data sets are generated, each containing 75 years of quarterly data for 3,000 firms. For each data set the above regressions are run over the last 35 years of data and then averaged. Coefficients and  $t$ -statistics are means over 1,000 simulations.

	(1)	(2)	(3)	(4)
Constant	0.00 (0.54)	0.01 (0.58)	0.03 (12.90)	0.12 (28.40)
$TQML_{t-1}$	0.16 (5.49)	0.16 (-4.98)		
$QML_{t-1}$	-0.17 (-12.56)	-0.17 (-12.47)		
$IDR_{t-1,t} - QML_{t-1}$			1.02 (85.33)	0.76 (69.77)
$\pi_{t-1}$			0.02 (1.38)	0.06 (2.23)
$\Delta\pi_{t-1}$		-1.20 (-7.68)		
$\pi_{t-1} \times (IDR_{t-1,t} - QML_{t-1})$			0.08 (10.78)	0.33 (24.25)
$R^2$	0.10 (36)	0.13 (36)	0.79 (36)	0.72 (31)
$N$	3,000	3,000	3,000	3,000

$$QML_t - QML_{t-1} = h_0 + h_1 TL_{t-1} + h_2 QML_{t-1} + h_3 X_{t-1} + \epsilon, \quad (22)$$

where  $X_{t-1}$  represents other possible lagged regressors. A partial adjustment model predicts that  $h_1$  is positive and  $h_2$  is negative and, furthermore, that they are equal in absolute value. Coefficient  $h_2$  measures the speed of adjustment of leverage to its target level.

Not surprisingly, I find that leverage is mean reverting. A coefficient of  $-0.17$  indicates that the mean reversion of leverage is 17% per year. Fama and French (2002) report a similar mean reversion speed of 7 to 10% for dividend payers and 15 to 18% for nondividend payers, which they refer to as a “snail’s pace.” My firms may be better characterized as “crouching tigers:” most of the time firms do nothing to the level of their book debt, but when they do make changes it is by a large amount. Also, in line with the results reported by Fama and French (2002), the average slopes on lagged leverage are similar in absolute value to those on target leverage and are therefore consistent with the partial adjustment model.



Column 2 adds change in profits as an additional regressor. While the results are very similar to those of Fama and French (2002), the interpretation in the context of the present model is slightly different. In particular, while they suggest that this result shows that short-term variation in earnings is largely absorbed by debt, in the model developed here a change in profitability that affects the leverage ratio is due exclusively to persistence of its effect on firm value.

Columns 3 and 4 of Table VI report estimations of regressions of the change in the leverage ratio of the type studied by Welch (2004). The regression can be written as

$$\begin{aligned} QML_t - QML_{t-k} = & l_0 + l_1(IDR_{t-k,t} - QML_{t-k}) + l_2\pi_{t-k} \\ & + l_3\pi_{t-k}(IDR_{t-k,t} - QML_{t-k}) + \epsilon. \end{aligned} \quad (23)$$

The idea is that a significant coefficient on profitability,  $\pi_{t-k}$ , shows that profitability incrementally explains leverage after controlling for equity returns. If the cross term is significant, then profitability also helps to explain leverage adjustment.

The estimates in Table VI indicate that, once stock returns are controlled for, profitability loses most of its power in explaining leverage but is still able to account for the adjustment behavior of firms in the cross section. The latter result is similar to the finding of Welch (2004). Empirically, profitability is found to retain some explanatory power that could be due to its temporary component.

#### *D.4. Discussion and Extensions*

The main results of this paper emerging from the previous discussion are as follows. First, empirical hypotheses should be based on model properties in true cross-sectional dynamics. Second, the inability of standard cross-sectional tests to distinguish between the competing explanations of capital structure behavior suggests the importance of looking for other empirical tests. The cross-sectional test fails because (1) it considers all firms simultaneously irrespective of their position in the refinancing cycle and (2) it utilizes the same historical information for all firms despite the fact that firms differ in the starting points of their refinancing cycles. All future successful empirical tests have to satisfy these two conditions.

An empirical procedure that would resolve the first problem above is running cross-sectional regressions conditional on the decision to refinance. An example of such a procedure is a discrete choice model (Hovakimian et al. (2001)). Table VII reports the results of conducting an identical test to that run by Fama and French (2002), on two subsamples, namely, firms that refinanced by issuing more debt in the last year (active firms), and firms that did not change their book debt over the last year (passive firms). Firms that defaulted or sold assets over the same period are excluded. A number of important results emerge. First,

**Table VII**  
**Cross-Sectional Regressions on Subsamples**

The table reports the results of cross-sectional regressions on the level of the quasi-market leverage ratio, *QML*. Independent variables are profitability ( $\pi$ ), volatility of cash flows ( $\sigma$ ), bankruptcy costs ( $\alpha$ ), asset sale costs ( $q_A$ ), and restructuring costs ( $q_{RC}$ ). The Ref. Point column gives the results obtained by running the regression at the refinancing point. The FF column reports the results of benchmark Fama and French (2002) regression using the Fama–MacBeth (1973) empirical procedure on the whole sample. Column “Active” reports the results of the same regression run on the subsample of firms that restructured over the last year. Column “Passive” reports the results of the same regression run on the subsample of firms that did not restructure over the last year. For each data set the above regressions are run over the last 35 years of data and then averaged. Coefficients and *t*-statistics are means over 1,000 simulations.

	Ref. Point	FF	Active	Passive
Constant	0.24 (22.29)	0.62 (34.02)	0.60 (143.56)	0.63 (32.77)
$\pi$	5.88 (30.95)	−0.78 (−7.47)	−0.03 (−2.48)	−0.84 (−7.89)
$\sigma$	−0.78 (−91.22)	−0.40 (−26.79)	−1.09 (−106.80)	−0.37 (−24.77)
$\alpha$	−0.32 (−7.30)	−0.47 (−10.25)	−0.50 (−15.97)	−0.46 (−9.39)
$q_{RC}$	3.67 (3.54)	−3.28 (−3.09)	5.52 (7.47)	−5.55 (−4.90)
$q_A$	−0.17 (−10.94)	−0.29 (−16.53)	−0.22 (−18.90)	−0.30 (−16.54)
$R^2$	0.89 (1)	0.08 (35)	0.77 (35)	0.07 (35)
$N$	3,000	3,000	346	2,606

for the subsample of active firms, the cross-sectional regression has almost the same degree of explanatory power as the refinancing-point regression (in which all firms are active by construction). Second, asset volatility and restructuring costs have a larger magnitude relative to the refinancing-point regression. This is because a set of active firms in the dynamic economy is *not a random selection* from the set of all firms. Firms with lower volatility and restructuring costs are represented in the subsample disproportionately. Third, and for this study most importantly, profitability, while still slightly negative, almost loses its explanatory power. Changing the firm-specific characteristics can change the direction of the profitability effect (see Section III). The cross-sectional regression on the active subsample resolves the problem of sample contamination with passive firms but it still uses only the information over the very short period, not resolving the problem that firms restructure at different times. This result is very close to the empirical result on profitability by Hovakimian et al. (2001).

Another empirical procedure is to use the duration model (Leary and Roberts (2005)) with the estimation of the hazard function of refinancing depending on all information in the current refinancing cycle. Applying the same method to

the simulated data of my model (unreported), two interesting results emerge. First, the model replicates the hump-shaped hazard function for fixed costs. The economic intuition behind the model can explain the rationale behind the hump-shaped form of the hazard rate. In the presence of fixed costs, firms optimally wait before restructuring again (this explains an initial increase in the hazard rate). However, the firms that wait too long are likely to be the firms whose fortunes deteriorated substantially and thus, conditional on waiting long enough, the probability of restructuring in the next period is reduced. The second interesting finding is that an increase in profitability shifts the hazard curve up and thus increases the probability of refinancing. In the pecking order world, for example, we would expect a different sign from that found by Leary and Roberts (2005). Thus, the duration model has the potential to distinguish various theoretical models.

### III. Robustness Tests

In this section, I describe the results of a number of robustness tests designed to investigate the extent to which the results above are sensitive to changes in parameter values and estimation procedure. The tests fall into two categories. First, using the benchmark data set, I investigate whether the results are influenced by the way in which the sample is constructed. In particular, outliers in the simulation of the evolution of firm asset values may have an undue influence. Second, I study whether perturbing the parameters or the model features has a significant impact on the results. For each robustness test I redo the whole analysis but, to keep the computations within practical bounds, the results are averaged over 50 simulated economies.

The key question is whether the main results of the paper survive the robustness tests. These include: (1) The relation between the average level of leverage at refinancing points and in a dynamic economy; (2) the average slope of the leverage–profitability relationship; (3) the results relating to Welch's (2004) finding on capital structure and stock returns, and (4) the degree of mean reversion. To save space, Table VIII reports only a summary of some of the main results.

The evolution of a dynamic economy leads to some outliers. While there is no measurement error in my benchmark data set, an empiricist using the data generated by any simulated economy might be concerned that some observations dominate the results and therefore should be excluded. Following the approach used in the literature, I examine how the results are changed when: (i) The true volatility of firm cash flows is trimmed at the 5<sup>th</sup> and 95<sup>th</sup> percentile thresholds; (ii) in a dynamic economy, the time-series volatility in each year is estimated for each firm over the previous 5 years and estimates outside the 5<sup>th</sup> and 95<sup>th</sup> percentiles are excluded; (iii) in a dynamic economy, for each year firms whose profitability lies outside the 5<sup>th</sup> and 95<sup>th</sup> percentiles are excluded; (iv) firms that experience default over the previous 5 years are excluded. I find that none of these changes in procedure influence the main results in any significant way.

**Table VIII**  
**Robustness Tests**

The table summarizes the robustness tests. Column (i) reports the difference between average leverage in dynamics and at Ref. Point (See Table III). Column (ii) reports the average value of the profitability coefficient in Fama–French regressions (Table IV). Column (iii) reports the average value of  $IDR_{t-1,t}$  (Table V). Column (iv) reports the average value of the mean-reversion coefficient  $f_2$  (Table VI). The tests are as follows: (1) Adjustment cost  $q_{RC} = 0.05\%$  for all firms; (2)  $q_{RC} = 0.35\%$  for all firms; (3)  $q_{RC} = 0.20\%$  for all firms; (4)  $q_{RC}$ 's distribution is shifted to the right by 0.15%; (5) Personal tax on interest income  $\tau_i = 25\%$ ; (6)  $\tau_i = 40\%$ ; (7) Interest rate  $r = 0.07$ ; (8)  $r = 0.03$ ; (9) Asset risk premia are 0.09%; (10) Asset risk premia are 0.04%; (11) Volatility of systematic component  $\sigma_S = 0.18\%$  (mean of total volatility,  $\sigma$ , is 0.299 with std. dev. of 0.106); (12)  $\sigma_S = 0.18$  ( $\sigma_I$  is multiplied by  $0.9 \times (\sigma_I - 0.05)$  so that  $\sigma$  has mean and std. dev. similar to the benchmark case); (13)  $\sigma_S = 0.05$  (mean of  $\sigma$  is 0.23 with std. dev. of 0.106); (14)  $\sigma_S = 0.05$  ( $\sigma_I$  is multiplied by  $0.93 \times (\sigma_I + 0.043)$  so that  $\sigma$  has mean and std. dev. similar to the benchmark case); (15)  $\sigma_S = 0.0$  ( $\sigma_I$  is multiplied by  $0.91 \times (\sigma_I + 0.055)$  so that  $\sigma$  has mean and std. dev. similar to the benchmark case); (16)  $\sigma$  has a mean of 0.35 and std. dev. of 0.14 by multiplying original volatility by 1.37; (17)  $\sigma$  has a mean of 0.15 and std. dev. of 0.06 by multiplying original volatility by 0.59; (18)  $\sigma$  is 0.247 for all firms (by taking the mean of  $\sigma_I$  and  $\beta$ ); (19)  $\beta$  distribution is wider by 50% (so that  $\sigma$  has a mean of 0.288 and std. dev. of 0.104); (20)  $\beta$  distribution is wider by 50% ( $\sigma_I$  shifts to the right by 0.05 so that  $\sigma$  has mean and std. dev. similar to the benchmark case); (21) All firms have the same firm-specific parameters, which are equal to the mean of the corresponding parameters; (22)  $\tau_l = 0$  for all firms; and (23) net payout ratio is 0.035 for all firms. Each test is run for 3,000 firms and 50 economies. In each test, other parameter values and empirical procedures are unchanged.

Test	Description	(i)	(ii)	(iii)	(iv)
1	$q_{RC} = 0.05\%$ for all firms	0.13	−0.62	1.02	−0.16
2	$q_{RC} = 0.35\%$ for all firms	0.10	−0.61	1.04	−0.17
3	$q_{RC} = 0.20\%$ for all firms	0.11	−0.69	1.03	−0.17
4	$q_{RC}$ : distribution is shifted to the right	0.09	−0.53	1.04	−0.17
5	$\tau_i = 25\%$	0.14	−0.60	1.02	−0.22
6	$\tau_i = 40\%$	0.06	−0.72	1.05	−0.12
7	$r = 0.07$	0.08	−0.43	0.99	−0.18
8	$r = 0.03$	0.14	−0.81	1.06	−0.16
9	Asset risk premia are 0.09%	0.08	−0.45	0.99	−0.17
10	Asset risk premia are 0.04%	0.14	−0.70	1.07	−0.17
11	$\sigma_S = 0.18$ (and thus $\sigma$ is larger)	0.13	−1.07	1.06	−0.17
12	$\sigma_S = 0.18$ (and $\sigma_I$ is smaller so that $\sigma$ does not change)	0.11	−0.64	1.07	−0.15
13	$\sigma_S = 0.05$ (and thus $\sigma$ is smaller)	0.09	−0.29	1.00	−0.17
14	$\sigma_S = 0.05$ (and $\sigma_I$ is larger so that $\sigma$ does not change)	0.11	−0.39	1.01	−0.17
15	$\sigma_S = 0$ (and $\sigma_I$ is larger so that $\sigma$ does not change)	0.11	−0.33	1.01	−0.17
16	$\sigma$ has a mean of 0.35	0.09	−0.94	1.03	−0.12
17	$\sigma$ has a mean of 0.15	0.12	−0.12	1.00	−0.31
18	$\sigma = 0.247$ for all firms	0.09	−1.32	1.02	−0.20
19	$\beta$ has a wider distribution (so that $\sigma$ increases)	0.12	−0.79	1.04	−0.17
20	$\beta$ has a wider distribution ( $\sigma_I$ is changed so that $\sigma$ is the same)	0.10	−0.48	1.05	−0.16
21	All firm-specific parameters equal to their cross-sectional means	0.10	−0.80	1.00	−0.16
22	No loss of tax shelter, $\tau_l = 0$	0.19	−0.89	1.02	−0.20
23	Net payout ratio is 0.035 for all firms	0.12	−0.66	1.01	−0.17

The next tests examine the dependence of the results on changes in the parameters. First, for each exogenous parameter that varies across firms, I consider five cases. For the first two, the distribution of the parameter is identical to the benchmark case except that its mean is changed; in one case increased and in the other decreased. In the remaining three cases, the parameter value is set equal across firms at (i) the upper boundary of the benchmark distribution, (ii) the lower boundary, and (iii) a value equal to the mean in the base case. Again I find that, qualitatively, the main results are unchanged. However, changing the volatility parameters does result in noticeable changes in the cross-sectional distribution of leverage. For a hypothetical sample of firms without access to public debt markets volatility is higher (Faulkender and Petersen (2006)) and test 16 shows that the cross-sectional result on profitability is stronger. Under some parameter values, empirical estimates of profitability are less likely to be obtained by the model. The coefficients on the implied debt ratio and mean reversion are more stable. An important observation is presented by test 15 in Table VIII, where the distribution of total volatility is similar to the benchmark case but there is no systematic component. As discussed above, the absence of a systematic component leads to a decrease in the magnitude of the profitability–leverage relation. Test 15 provides a quantitative assessment, with a coefficient of  $-0.33$  as opposed to  $-0.78$  in the benchmark case. In addition, I consider the case with identical firm-specific parameters, where the only dynamic effect comes from random changes in value. Test 21 demonstrates that this does not change any results. To study the importance of some model features, I consider two cases of the benchmark model: (i) without the loss of the tax shelter ( $\tau_l = 0$ ), and (ii) with constant net payout ratio equal to 0.035. Tests 22 and 23 show that the qualitative results are unchanged.

Third, I investigate the effect of changes in macroeconomic and tax parameters. Unsurprisingly, decreasing the tax advantage to corporate debt results in lower leverage in the economy. One result not shown in Table VIII is that a decrease in  $\tau_i$  from 0.35 to 0.25 lowers the average market leverage ratio from 0.36 to 0.28. A decrease in  $\tau_i$  also leads to a substantial increase in the difference between the average leverage ratio in dynamics and at the refinancing point.

Finally, I consider the effect of measurement errors. Erickson and Whited (2000) find that the market-to-book ratio contains a great deal of measurement error. Since, in the simulated model, the market-to-book ratio and profitability are highly correlated, I introduce a classical error-in-variables problem by adding a stochastic component to the evolution of profitability, similar to Gomes (2001, eq. (35) on p. 1281). Note that these measurement errors are assumed not to affect the optimal decisions by firms. Similar to Gomes (2001), I find (unreported in the table) that the coefficient on profitability changes substantially, from  $-0.78$  to  $-0.33$ . A similar perturbation of the book value of assets only changes the coefficient from  $-0.78$  to  $-0.71$ .

Overall, the results appear to be quite robust with respect to changes in firm-specific and environmental parameters, and to changes in empirical procedure.

This applies particularly to the cross-sectional results, which are also the most important.

#### **IV. Concluding Remarks**

This paper is the first to describe a methodology for deriving the quantitative and qualitative predictions of capital structure theories in a dynamic economy with infrequent adjustment. Using a model of dynamic optimal capital structure, I generate data that structurally resemble data used in empirical studies. In this way, the method allows us to compare the predictions of a capital structure model in “true dynamics” both to the findings of the empirical literature and to the comparative statics predictions of the same model. In particular, it enables us to provide greater insight into the qualitative aspects of the cross sectional properties of leverage. The main findings of the paper are that (1) the properties of leverage in the cross section in true dynamics and in comparative statics at refinancing points differ dramatically, and (2) the model gives rise to data that are consistent with a number of empirical results and that, using methodologies commonly employed in the literature, may lead to rejection of the model. These findings highlight the need for further research in this area.

There are two principal directions in which the framework developed here could most usefully be extended. First, because the dynamics of financing decisions have such a profound influence on the empirical properties of the cross section, competing theories of capital structure—beyond the dynamic trade-off theory—ought to be developed in fully dynamic form. Some first attempts have been made. Dasgupta and Sengupta (2002), for example, develop a model with moral hazard in which, interestingly, dynamic interaction leads to another explanation for a positive relation between leverage and profitability. Nevertheless, development of alternative dynamic models that lead to quantitative predictions is still a subject for future research.

Second, a proper study of the evolution of capital structure requires a model that combines both dynamic capital structure decisions and real investment. Examples of capital structure models with endogenous investment are Brennan and Schwartz (1984), Hennessy and Whited (2005), and Titman and Tsyplakov (2003). Berk et al. (1999) provide another excellent basis for studying real investment, enabling researchers to analyze book values in addition to market values, while the model developed here contributes to dynamic capital structure. The modeling approach of firm behavior in Berk et al. (1999) is both richer than mine in some areas and less rich in others. In particular, they are able to analyze a wider spectrum of questions by considering separately existing assets in place and future growth opportunities. However, their firms are myopic since the fact that investment projects are assumed independent, combined with a complete lack of any financial policy, means that in taking investment decisions, a firm does not take into account the evolution of its assets over time. Research that combines these two strands is likely to be a fruitful avenue for future research in capital structure, and more generally, corporate finance.

## Appendix: Details of Simulation Analysis

The process for  $\delta$  is discretized using the following approximation:

$$\delta_t = \delta_{t-\Delta t} e^{(\mu_A - \frac{\sigma^2}{2})\Delta t + \sigma\sqrt{\Delta t}z_t}, \quad (\text{A1})$$

where  $\Delta t$  is one quarter,  $z_t$  is a standard normal variable, and  $\mu_A$  is the growth rate of the net payout ratio under the physical measure. The benchmark simulation is for 300 quarters and 3,000 firms. Note that while I discretize the model for the purpose of simulation, firms still operate in a continuous environment. In particular, firms sometimes “overshoot” boundaries and make their financial decisions at times different from the prescribed optimal times. Unreported robustness checks show that increasing the frequency of observations does not change the results.

To minimize the impact of initial conditions, I implement the following ad hoc procedure to choose the number of observations that will be dropped. I simulate the panel data set for 3,000 firms with the benchmark set of parameters, in the absence of a systematic shock, 250 times. For each economy the average leverage ratio is calculated. I then estimate the rolling sum of the first differences in average leverage ratios (quarter by quarter) over the last 10 quarters. The stopping rule for this variable is to be less than 0.5% in absolute magnitude (for comparison, the average value of this variable in the first 10 quarters is 5%), at which point the economy is defined as converged to its steady state. The resulting distribution of steady-state stopping times across all economies has a mean of 30 quarters, a 95<sup>th</sup> percentile value of 50 quarters, and a maximum of 76 quarters. For a conservative estimate I double the maximum. Since this procedure is largely ad hoc, I check the result by simulating 20 economies for 1,000 quarters and confirm that there is no difference in the average leverage ratio behavior for the last 900 quarters by investigating rolling sums over the entire period.

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